Waveforms

Many of the voltage and current we shall encounter change their value with time. A graph which shows this variation with time is called a waveform. The most important waveforms are: the dc, the ac and the step function.

The constant or “dc” waveform

The constant or “dc” waveform (for “direct current”) is produced by constant or “dc” sources that keep its value unmodified with time (that is the simplest waveform).

The step function

The step function is a constant waveform that is “turned on” at some instant of time. The step function is:

$$v = A_0 \cdot u_{-1}(t - t_0).$$

The symbolic notation $u_{-1}(x)$ means: $u_{-1}=0$ for $x<0$ and $u_{-1}=1$ for $x>0$.

The sinusoidal or “ac” waveform

The sinusoidal or “ac” waveform (for “alternating current”) is a waveform that continually alternates between a peak positive value and a peak negative value, following a sine (or cosine) function. An example is given for a current sine wave.
The magnitude of the peak value is called **the amplitude** of the wave: e.g. I_{pk}.
The sinusoidal waveform is periodic, repeating a basic pattern over and over. The time for a full cycle is denoted T, that is **the period** of the wave.
The frequency specifies how many full cycles the waveform goes through in one second and it is measured in Hertz: $f = \frac{1}{T}$ [Hz].
The angular frequency, measured in radians per second, is: $\omega = 2\pi \cdot f$ [rad/sec].
The root-mean-square (rms) amplitude of the waveform for a sine wave is:

$$I_{rms} = \frac{I_{pk}}{\sqrt{2}}.$$

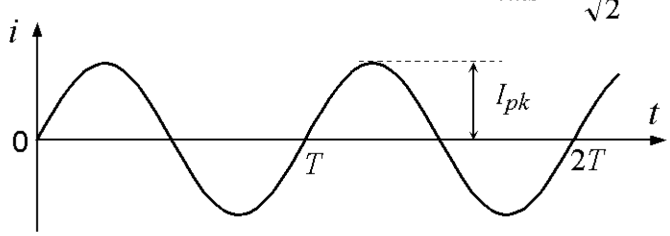


Figure – Sinusoidal signal;
Parameters:
I_{pk} – peak voltage,
T – signal period.

The sine waveform function is: $i = I_{pk}\sin(2\pi \cdot f) = \sqrt{2} \cdot I_{rms}\sin(2\pi \cdot f)$.

Pulse sequence

Consist of a sequence of positive going pulses. It is used in digital systems and computers; the principal features of these digital waveforms is the “On” and “Off” nature of the pulse sequence at some precise instant of times. The amplitude is not critical as long as it falls within a rather wide predetermined range.

Notation convention

To distinguish the different type of time-varying waveforms, a convention has been widely adopted regarding the use of capital or lower-case letters for network variables (v or i).

Waveform / signal type	Variable	Subscript	Examples
General network variable	Lower case	Upper case	v_A, i_C;
"dc" component of the waveform	Upper case	Upper case	V_A, I_C;
Incremental ("ac") component of wave	Lower case	Lower case	v_a, i_c;
"rms" amplitude	Upper case	Lower case	V_a, I_c;

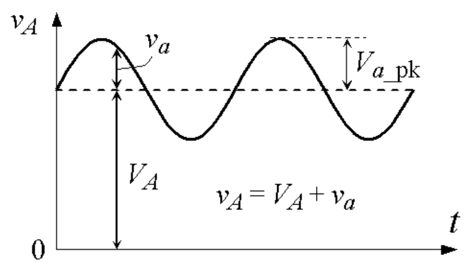


Figure – notation convention;
Sine signal v_a added to the bias voltage V_A.

Superposition of Independent Sources

A way of breaking up problems containing more independent sources into several smaller problems is called "superposition". Superposition means to determine the response of each independent source, one at a time, assuming that all other independent sources are set to zero (suppressed) and then sum the results to get the total response.
The superposition theorem is a consequence of the linearity of Kirchhoff laws applied to linear circuits. **Superposition can be used ONLY in LINEAR networks!**

Source Suppression

If a voltage source is suppressed or set to zero, it means that any amount of current can flow and no voltage will result. This is called "short-circuit" (zero resistance circuit – a wire). If the current is zero regardless to the voltage, we have an "open-circuit" (infinite resistance, nothing connected). No current can flow in an open-circuit.

Example for superposition method

For a circuit with two independent sources, as the one in the next figure, compute the currents that flow in the resistances.

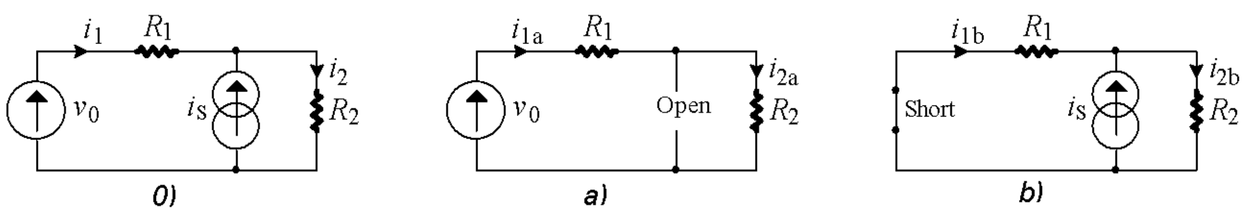


Figure – *Example for the superposition method: 0) the actual circuit; a) the circuit with the current source suppressed; b) the circuit with the voltage source suppressed.*

For case *a)* (i_S set to zero) the circuit is a voltage divider, the resistors appears in series with the voltage source and both must carry the same current: $i_{1a} = i_{2a} = \dfrac{v_0}{R_1 + R_2}$.

For case *b)* (v_0 set to zero) the circuit is a current divider, the current divider rule applied to both branches of the circuit gives: $i_{1b} = -\dfrac{R_2}{R_1 + R_2} i_S$, $i_{2b} = \dfrac{R_1}{R_1 + R_2} i_S$.

Finally, adding the effects of the two sources we get:

$$i_1 = i_{1a} + i_{1b} = \frac{v_0 - i_S R_2}{R_1 + R_2}, \quad i_2 = i_{2a} + i_{2b} = \frac{v_0 + i_S R_1}{R_1 + R_2}.$$

Using KVL and KCL on the entire network, one will get the same results, but with more algebraic manipulations.

The Semiconductor Diode

The semiconductor diode is the device that conducts current in one direction and blocks current in the other direction. That is the main characteristics of the diode.
A semiconductor diode is a two terminal device containing a single *p-n* junction. The two terminals of the diode are the anode and the cathode. The arrow portion of the diode symbol indicates the direction of the forward current (and the direction of forward voltage).

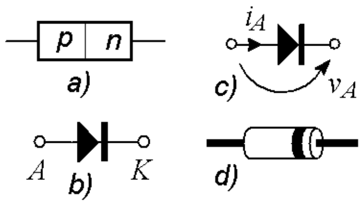


Figure – Semiconductor Diode: *a)* a *p-n* junction, *b)* its symbol is an arrow pointing from anode to cathode; *c)* the arrow indicates the direction of the forward current and voltage and *d)* the cathode is marked.

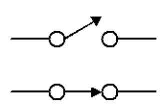
When the anode is positive with respect to the cathode the diode is forward biased and the diode carries a forward current. When the anode is negative with respect to the cathode, the diode is reversed biased and the reversed current is a small saturation current.

Diode Models

The ideal diode

The simplest way to visualize diode operation is to think of it as a switch. When forward biased (positive source pin on the anode terminal) the diode acts as a closed (or ON) switch and when reverse biased it acts as an open (or OFF) switch. The resulting circuit element is called the "ideal diode" and is characterized by equations:

$$\begin{cases} i_A = 0 & \text{for } v_A < 0 \quad \text{(open switch)} \\ v_A = 0 & \text{for } i_A > 0 \quad \text{(closed switch)} \end{cases}.$$



The symbol used for the ideal diode element and the current-voltage (i-v) characteristics are indicated in the next figure.

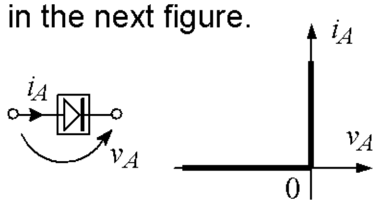


Figure – The symbol and i-v characteristics of the ideal diode.

The constant-voltage-drop model

The forward-biased diode is represented as a closed switch in series with a small “battery” equal to about 0.7 V for silicon (and 0.3 for germanium). The positive end of the battery is toward the anode. The equivalent circuit and static characteristics are presented in the next figure; the equations for this model of the diode are:

$$\begin{cases} i_A = 0 & \text{pentru} \quad u_A < V_D \\ v_A = V_D & \text{pentru} \quad i_A > 0 \end{cases}.$$

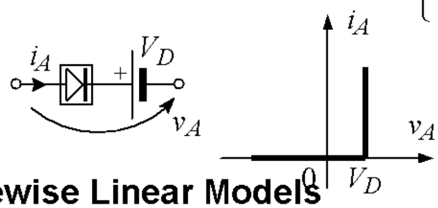


Figure – The symbol and i-v characteristics of the constant-voltage-drop diode model.

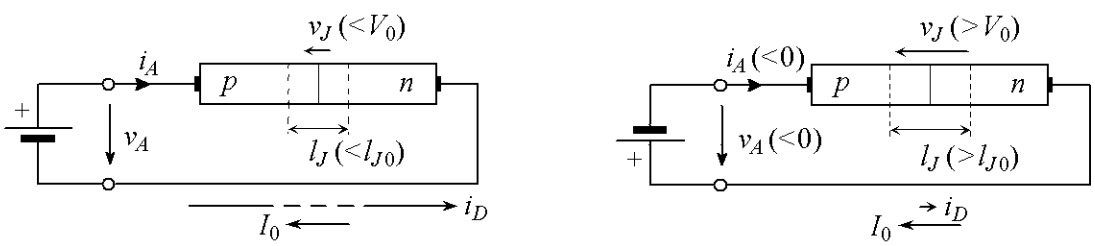
Piecewise Linear Models

A nonlinear curve that consists of strait line segments is said to be “piecewise linear”.
If a signal across its terminals swings only along one of the linear segments, then the device can be considered a linear circuit element. On the other hand, if signals swing past one or more of the break points in the characteristics, linear analysis is no longer possible.
…

Physical Operation of Diode

Here is a brief explanation of the physical diode. The semiconductor diode is basically a *p-n* junction. The p-n junction consists of a single semiconductor crystal doped so that part of it is n-type and the other part is p-type.
Because of the recombination process (produced by the concentration gradient) near the junction results:

- a carrier depletion region and
- a space charge region, that is an electrical field that opposes to diffusion that gives an internal voltage drop V_0; this voltage drop controls the diffusion current.



Diode forward bias

The diode is forward bias when the junction p-region (anode) is positive with respect to the n-region (cathode), v_A>0. The lower barrier voltage over the depletion region (v_J=$V_0 - v_A$) allow an exponentially increase of the diffusion current (i_D).

Diode reverse bias

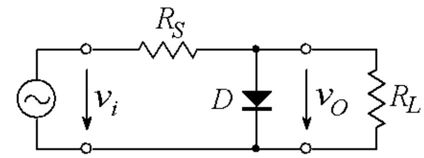
The diode is forward bias when v_A<0. The greater barrier voltage over the depletion region (v_J=$V_0 + v_A$) decreases the diffusion current i_D to 0. It remains only a very small drift current (produced by the minority carrier, I_0).

Diode Applications – Diode Limiters

Diode limiters are sometime used to clip off portion of signal voltages above or bellow certain levels; these circuits are called limiters or clippers.

Positive clipper

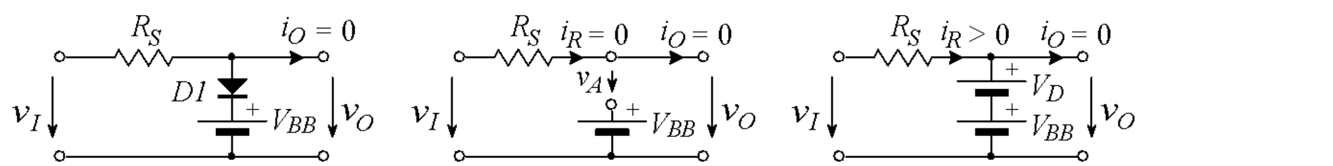
A diode circuit that limits the positive part of input voltage (also called a positive clipper) is presented in the next figure.



Circuit operation: Since the cathode is at the ground potential (0 V) the anode cannot exceed the threshold voltage V_D=0.7 V. Point A (diode anode) is limited to this 0.7 V value. When the input goes back below 0.7 V, the diode reverse biases and appears as an open-circuit. The output voltage is:
If R_S is small compared to R_L then $v_O = v_I$.
If the diode is turned around, the negative part of the input is clipped off (bellow $-V_D = -0.7$ V). The positive and negative clippers can be combined. Such a circuit can be used to protect the amplifier inputs.

Adjustment of the Limiting Level

The level to which a signal voltage is limited can be adjusted by adding a bias voltage V_{BB} in series with the diode, as indicate din the next figure.



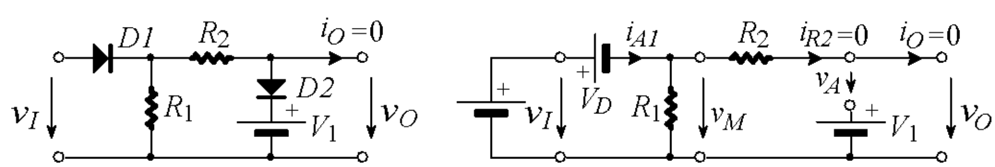
Circuit operation: The voltage level at point A must equal $V_{BB}+V_D$ before the diode to conduct. Once the diode begins to conduct, the voltage at point A is limited to $V_{BB}+V_D$ so that all input voltage above this level is clipped off. If the bias level is modified, the limiting level changes correspondingly.

Applications

S2 – P1. For the circuit in figure with V_D=0.7 V, V_1=2 V and $R_1 = R_2 = 1\,\text{k}\Omega$:

a) Derive and plot the transfer characteristic $v_O(v_I)$,
b) Derive and plot the output characteristic for: $v_i = V_p \sin(2\pi \cdot f) = 5\sin(100\pi)$.

Consider the constant-voltage-drop model for the diodes.



a) Circuit analysis:

A voltage source is implicitly assumed at the circuit input and it is represented in the equivalent circuit (upper right figure). V_1 voltage source is a passive one, it can not provide current because of diode $D2$. At the circuit output there is nothing connected (the output current being zero); it is assumed an ideal voltmeter connected at the output to measure the output voltage). The circuit contains two diodes and any of them can have two states (ON or OFF state); as a result there are four states possible. If both diodes are cut-off, the current through the diodes are: $i_{A1}=i_{A2}=0$. As long as diode $D1$ is cut-off the current through R_1 is zero: $i_{R2}=i_O+i_{A2}=0$, $i_{R1}=i_{A1}-i_{R2}=0$ and the median voltage is zero: $v_M=R_1\cdot i_{R1}=0$. On the other hand, the current through R_2 is also zero and as a result the output voltage is zero: $v_O=v_M-R_2\cdot i_{R2}=0$.

The diode $D1$ is blocked for $v_{A1}<V_D$; that means: $v_{A1}=v_I-R_1\cdot i_{R1}=v_I<V_D$.

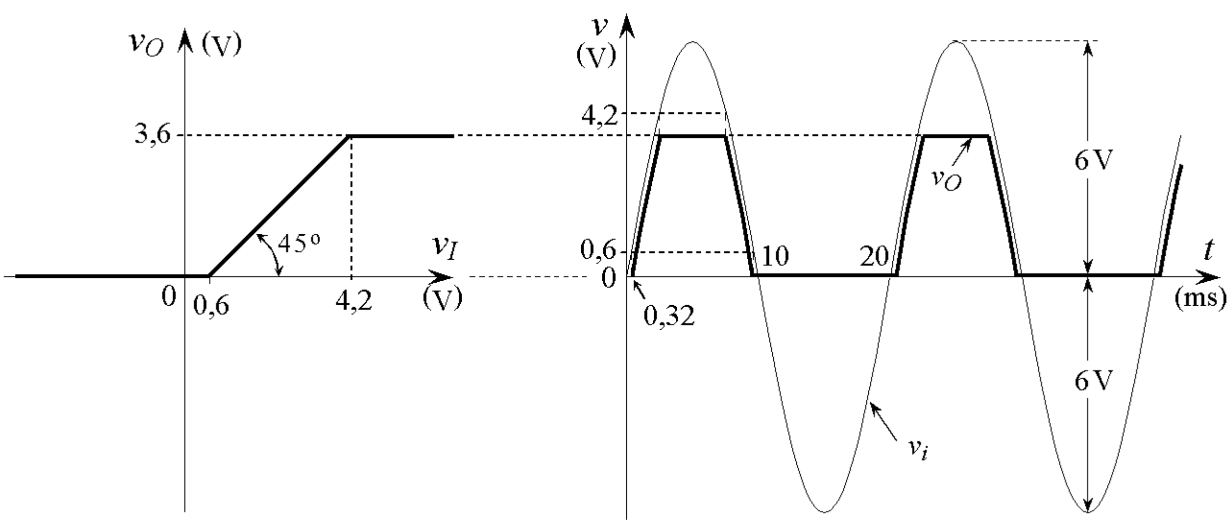
At the conduction limit for diode $D1$, the current through the diode is still zero and the voltage drop over R_1 is also zero: $v_{A1}=v_I-R_1\cdot i_{R1}=v_I=V_D$. For $v_I>V_D$ the diode is in conduction state, $u_{A1}=U_D$ and the median voltage $v_M=v_I-v_{A1}$ becomes $v_M=v_I-V_D$. The voltage limiter with bias, R_2-$D2$-V_1, has v_M voltage at its input and v_O at the output. The diode $D2$ begin to conduct for $v_M=V_1+V_D=v_I-V_D$, that means $v_I=V_1+2V_D$. The circuit operation can be described by relations:

$$\begin{cases} v_O = v_M = 0 & \text{if} \quad v_I < V_D & \text{, both diodes are cut - off;} \\ v_O = v_I - V_D & \text{if} \quad V_D \le u_I < 2\cdot V_D + V_1 & \text{, } D1 \text{ conducts, } D2 \text{ is cut - off;} \\ v_O = V_D + V_1 & \text{if} \quad v_I \ge 2\cdot V_D + V_1 & \text{, both diodes conducts .} \end{cases}$$

The forth combination, with $D1$ cut-off and $D1$ in conduction, can not be realized for this circuit: if $D1$ is cut-off, $v_M=0$ and in this case $D2$ is also cut-off.

If one considers the voltages defined for this application the analytical description of transfer characteristics is given by equations:

$$\begin{cases} v_O = 0 & \text{if} \quad v_I < 0{,}6\text{V} \\ v_O = v_I - 0{,}6\text{V} & \text{if} \quad 0{,}6\text{V} \le v_I < 4{,}2\text{V} \\ v_O = 3{,}6\text{V} & \text{if} \quad v_I \ge 4{,}2\text{V} \end{cases}$$



a) The transfer characteristics, *b)* The input and output voltage waves

The transfer characteristics is represented in the previous figure (left side) and it is a direct representation of the upper equations (a piecewise linear function).
To get the output waves, one has to replace the input equation in the previous relations.
The diode conduction angles, and the corresponding times, can be computed as follows:

$$v_{i1} = 6\sin\alpha_1 = 6\sin\left(100\pi\cdot t_1\right) = 0{,}6 \Rightarrow \alpha_1 = \arcsin\frac{0{,}6}{6} = 0{,}1\,\text{rad},\quad t_1 = \frac{0{,}1}{100\pi} = 0{,}32\,\text{ms}$$

$$v_{i2} = 6\sin\alpha_2 = 6\sin\left(100\pi\cdot t_2\right) = 4{,}2 \Rightarrow \alpha_2 = \arcsin\frac{4{,}2}{6} \cong 0{,}78\,\text{rad},\quad t_2 \cong \frac{0{,}78}{100\pi} = 2{,}5\,\text{ms}$$

One can see that the output wave can be obtained by translating down the input wave by $0{,}6\ \text{V}$ and clip-off the voltages greater than $3{,}6\ \text{V}$.

S2 – P2. For the circuit in figure, assuming an ideal diode, $R_1 = 1\,\text{k}\Omega$ and $R_2 = 2\,\text{k}\Omega$:

I. Specify the range of v_I for which the diode is ON (short circuit) or OFF (open circuit);
II. Sketck and dimension the input characteristic $i_I(v_I)$ and
III. Sketch and dimension the transfer characteristic $v_O(v_I)$ for the two cases:

a) $V_B = 0$ and *b)* $V_B = 2\ \text{V}$.