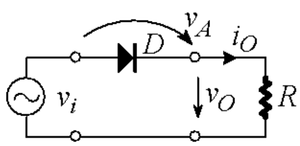
Diode Applications

Because of their ability to conduct current in only one direction, diodes are used in rectifier circuits. Rectification is the process of converting *ac* to pulsating *dc*.

Half-wave Rectifier

The *ac* source is connected to the load through a diode, as indicated in the next figure.



The circuit will be analyzed, firstly, with the ideal diode model.

When the input wave goes positive, the diode is forward bias and conducts current to the load resistance R_L; The ideal diode may be replaced by a short-circuit.

When the input voltage goes negative, during the second half of the cycle, the diode is reversed biased. There is no current, so the voltage across the load resistor is zero.

VKL gives: $v_O = v_I - v_A$.

1. For $v_I > 0$, the current is $i_O > 0$ and $v_A = 0$,
2. For $v_I < 0$, the current is $i_O = 0$ and $v_O = R_L\, i_O = 0$.

For a sine wave input, $v_i = V_p \sin \omega t$, the maximum reverse voltage: $v_{A\mathrm{Max}} = -V_p$, gives the peak inverse voltage parameter of the diode: $PIV = |v_{A\mathrm{Max}}| = V_p$.

Only the positive half-cycle of the *ac* input voltage appears across the load, making the output a pulsating *dc* voltage. Therefore, this process is called “half-wave rectification”.

The average (dc) voltage (and current) of the half-wave output signal is determined by finding the area under the curve over a full cycle (and by Ohm’s law):

$$V_O = \frac{1}{2\pi}\int_0^{\pi} \sqrt{2}\cdot V_i \sin \omega t \,\mathrm{d}\,\omega t = \frac{\sqrt{2}\cdot V_i}{\pi} = \frac{V_p}{\pi}, \quad I_O = \frac{V_O}{R_L}$$

Barrier potential effect

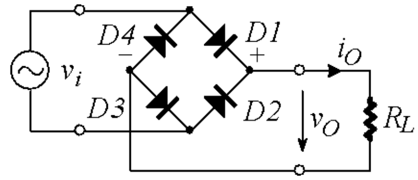
When the diode voltage drop is taken into account, during the positive half-cycle, the input voltage must overcome this voltage drop before the diode becomes forward biased; This result in a peak output value that is V_D ($=0.7$ V) less than the peak value of the input. The current is positive: $i_O > 0$ for $v_A > V_D$; that gives $v_O = v_I - V_D$ and an average output:

$$V_O = \frac{1}{2\pi}\int_{\alpha}^{\pi-\alpha} \left(V_p \sin \omega t - V_D\right)\mathrm{d}\,\omega t = \frac{V_p}{\pi}\cos\alpha - V_D\,\frac{\pi - 2\alpha}{2\pi} \quad \text{with} \quad \alpha = \arcsin\frac{V_D}{V_p}.$$

For $\alpha \ll \pi$ (obtained for $V_D \ll V_p$) we get $\cos\alpha \cong 1$ and a more convenient relationship (and enough precise for practical purposes): $V_O \cong \dfrac{V_p}{\pi} - \dfrac{V_D}{2}$.

The effect of barrier potential of the diode can be neglected only when the peak value of the input is much greater than this barrier potential (practically greater than about 10V).

Full-wave Bridge Rectifier



The bridge rectifier is presented in the next schematic; it consists on four diodes connected to provide same direction for the output current/voltage for both input polarities of the ac input source.
Let's consider first the ideal diodes model.
When the input is positive the current will flow through D_1, R_L and D_3, while at the same time D_2 and D_4 will be reverse biased. When v_i reverses polarity and becomes negative, conduction will result through D_2, R_L and D_4, with D_1 and D_3 reversed. The current in R_L is unidirectional supplied by either D_1 or D_2 and removed by either D_3 or D_4.
Bridge rectifiers are commercially available as 4 terminal packages. The source and the load are assumed to share no common terminals.
The average (dc) voltage (and current) of a full-wave output is twice the ones of the half-wave rectifier (the area under the curve for one signal period doubles):

$$V_O = \frac{2}{2\pi}\int_0^{\pi} \sqrt{2}\cdot V_i \sin \omega t \, \mathrm{d}\omega t = \frac{2V_p}{\pi}, \quad I_O = \frac{V_O}{R_L}$$

Two diodes are always in series with the load resistor during both positive and negative half cycles. If the diode voltage drops are taken into account, the output voltage is:

$$v_O = |v_I| - 2V_D$$

and the average output voltage is:

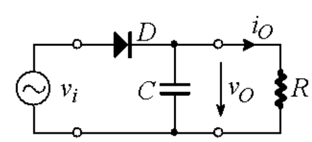
$$V_O = \frac{1}{\pi}\int_{\alpha}^{\pi-\alpha} \left(V_p \sin \omega t - 2V_D\right)\mathrm{d}\omega t = \frac{2V_p}{\pi}\cos\alpha - 2V_D\frac{\pi - 2\alpha}{\pi} \quad \text{with} \quad \alpha = \arcsin\frac{2V_D}{V_p}.$$

If $\alpha << \pi$ (obtained for $2V_D << V_p$) we get $\cos\alpha \cong 1$ and a more convenient relationship (and enough precise for practical purposes): $V_O \cong \frac{2V_p}{\pi} - 2V_D$.

Rectifier Filters

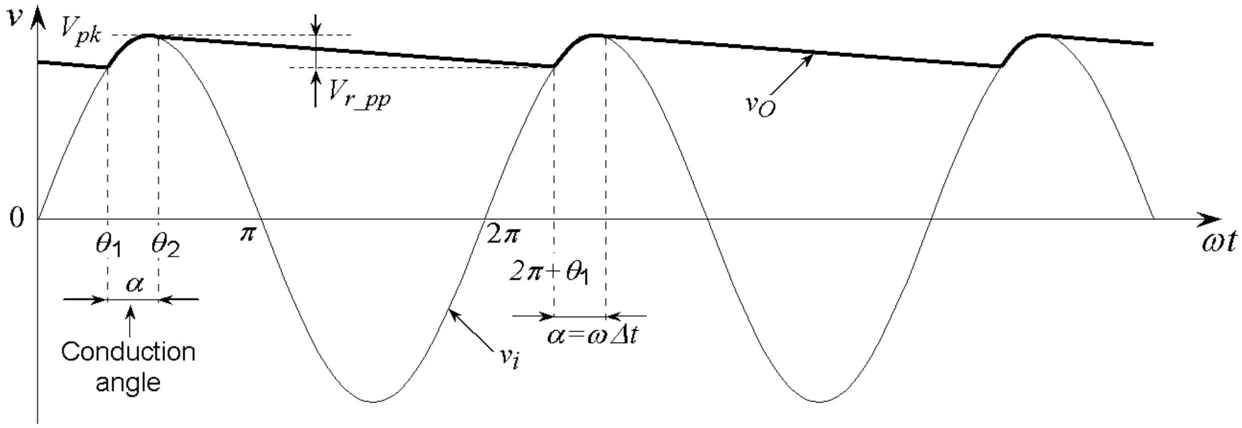
The uses for a pulsating dc are limited to charging batteries, running dc motors and few other applications. For electronic systems, the pulsating output of the rectifier must be filtered to virtually eliminate the large voltage variations. The most common filter uses a capacitor in parallel to the load.

Capacitor Input Filter



The half-wave rectifier with a capacitive-input filter is presented in the next figure. An ideal input source and an idea diode will be considered first.
The capacitor is charged to the input voltage peak. When the input begins to decrease (below the peak) the capacitor retains its charge and the diode becomes reverse biased. During the remainder part of the cycle the capacitor is discharged by the load at a rate determined by the RC time constant. The larger the time constant the less the capacity will discharge. During the first quarter of the next cycle the diode will become forward biased, when the input voltage exceeds the capacitor voltage.

The capacitor quickly charges at the beginning of the cycle (during the diode conduction angle, α) and slowly discharges after the positive peak for the rest of the cycle (when the diode is reverse biased, diode off angle, $\pi-\alpha$). The variation of the output voltage (due to the capacitor charging and discharging) is called ripple. The smaller the ripple, the better the filter is.



The ripple factor is the ration of the *rms* ripple voltage to the *dc*, average value of the output voltage, indicates the effectiveness of the filter.
During the diode off time interval the output voltage is given by the capacitor discharging on the load resistance (with V_p the initial capacitor voltage):

$$v_O = V_p \exp\left(-\frac{t}{RC}\right).$$

At the end of discharge interval, the output voltage can be approximated to:

$$v_{O\min} = V_p - V_{r(pp)} = V_p \exp\left(-\frac{T-\Delta t}{RC}\right) \cong V_p \exp\left(-\frac{T}{RC}\right).$$

This approximation is valid for normal case with the diode conduction time much less than the signal period ($\Delta t < T$; the off time approximated by the period: $T-\Delta t \approx T$).
Since $RC >> T$, we can approximate the function with the first term of Taylor series:

$$\exp\left(-\frac{T}{RC}\right) \cong 1 - \frac{T}{RC}.$$

The output voltage and peak-to-peak ripple voltage can be derived as:

$$v_O \cong V_p \cdot \left(1 - \frac{T}{RC}\right) = V_p - V_{r(pp)},\ V_{r(pp)} \cong V_p \cdot \frac{T}{RC}.$$

The ripple waveform can be approximated to a triangular wave; for a triangular wave: $U_{\mathrm{vf}}/U_{\mathrm{ef}} = \sqrt{3}$ and the ripple factor is:

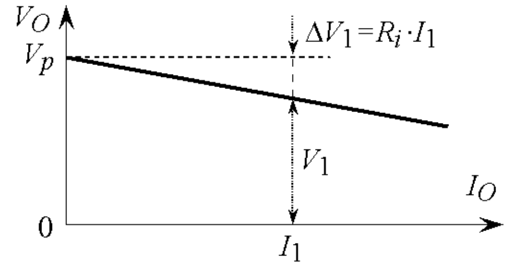
$$r = \frac{V_r}{V_O} = \frac{V_{r(pp)}}{2\sqrt{3}\cdot V_O} \cong \frac{T}{\sqrt{3}\cdot(2RC - T)} \quad \left(\cong \frac{1}{2\sqrt{3}} \cdot \frac{T}{RC}\right).$$

The *dc* (average) output voltage is:

$$V_O = V_p - \frac{V_{r(pp)}}{2} = V_p - V_p \frac{T}{2RC} = V_p - \frac{V_p}{R}\frac{T}{2C} \cong V_p - I_O \cdot \frac{T}{2C}.$$

(To keep the formula simple the *dc* load current was approximated with the ratio of the peak voltage to the load resistance.)

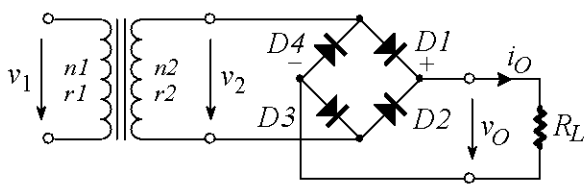
The result represents the rectifier output characteristics (for the average values), as indicated in the next figure, with an equivalent internal resistance: $R_i = T/2C = \pi X_C$.
For the full-wave rectifier the signal period at the output is half of the input signal period; same relationship can be used to compute the ripple factor and the average output voltage.



Application

S3 – P1. The transformer used with a bridge rectifier supply 30 V rms for a 230 V input voltage. The windings resistances are: primary winding $r_1 = 70\,\Omega$ and secondary winding $r_2 = 3\,\Omega$. Compute the average value of the output voltage for a load resistance $R_L = 20\,\Omega$. Consider the diode voltage drop $V_D = 0.9$ V (the current being in the amp range),
The transformer equivalent resistance, seen in the secondary, can be computed as:

$r_T = r_2 + r_1\left(\frac{n_2}{n_1}\right)^2$ with n_2 / n_1 the turns ratio (secondary over primary turns no). This ratio is the same to the corresponding voltage ratio: $\frac{n_2}{n_1} = \frac{v_2}{v_1}$.

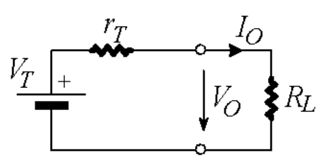


The rectifier can be seen as a *dc* power supply (Thevenin equivalent) from the load point of wiev. The Thevenin voltage is the *dc* output voltage (computed with the constant voltage drop diode model):

$$V_T = \frac{2 \cdot V_{2(pk)}}{\pi} - 2 \cdot V_D = \frac{2 \cdot \sqrt{2} \cdot V_{2(avg)}}{\pi} - 2 \cdot V_D = 0.9 \cdot 30 - 2 \cdot 0.9 = 25.2\,\text{V}$$

The resistive part of the Thevenin source consists on the resistance in the secondary loop; that is the transformer equivalent resistance seen in the secondary (to the rectifier):

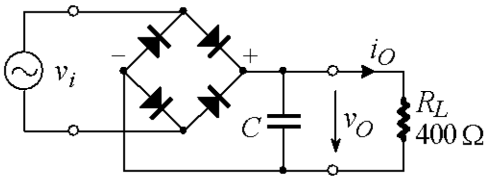
$$r_T = r_2 + r_1\left(\frac{n_2}{n_1}\right)^2 = r_2 + r_1\left(\frac{V_2}{V_1}\right)^2 = 3 + 70\left(\frac{30}{230}\right)^2 = 3 + 70 \cdot 0.017 = 4.19\,\Omega.$$



The dc equivalent circuit with transformer/rectifier replaced by the Thevenin equivalent source is basically a voltage divider that permits a direct computing of the output voltage:

$$V_O = \frac{R_L}{R_L + r_T} V_T = \frac{20}{20 + 4.19} 25.2 = 0.827 \cdot 25.2 = 20.8\,\text{V}$$

S3 – P2. The rectifier input voltage is a sine with a maximum value of 30 V. *a)* What is the dc load voltage for C=220 μF? What is the rms ripple voltage and the ripple factor?
b) Modify the capacitor to get a 1% ripple factor.



a) $\dfrac{V_{r(pp)}}{V_p} = \dfrac{T}{RC}$, $\quad V_{r(pp)} = \dfrac{T}{RC} V_p = \dfrac{0.01}{0.4k \cdot 0.22m} 30 = 3.41\,\text{V}.$

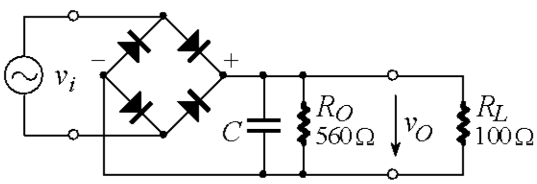
$$V_O = V_p - \frac{V_{r(pp)}}{2} = 30 - 1.7 = 28.3\,\text{V}, \qquad V_r = \frac{V_{r(pp)}}{2\sqrt{3}} = 0.984 \cong 1\,\text{V}.$$

$$r = \frac{V_r}{V_p} = \frac{1}{30} 0.033 = 3.3\,\%$$

b) $C = \dfrac{T}{R} \cdot \dfrac{V_p}{V_{r(pp)}} = \dfrac{T \cdot V_p}{2\sqrt{3} \cdot V_r \cdot R} = \dfrac{0.29}{r \cdot R} = \dfrac{0.29}{1 \cdot 400} = 0.725\,\text{mF} = 725\,\mu\text{F}.$

S3 – P2. Estimate the filter capacitor value in a full wave rectifier if:

- a resistence of 560 ohms resistance is measured between the output terminals,
- for a 100 ohm external load, one measured 15.4 V *dc* and 0.17 V *ac* (rms).



$$V_{r(pp)} = 2\sqrt{3} \cdot V_r = 2\sqrt{3} \cdot 0.17 = 0.59\,\text{V}.$$

$$V_p = V_O + \frac{V_r}{2} = 15.4 + \frac{0.59}{2} = 15.7\,\text{V}, \qquad R = \frac{R_L R_O}{R_L + R_O} = \frac{100 \cdot 560}{100 + 560} = 85\,\Omega.$$

$$\frac{V_{r(pp)}}{V_p} = \frac{T}{RC}, \qquad C = \frac{T}{R} \cdot \frac{V_p}{V_{r(pp)}} = \frac{0.01}{85} \frac{15.7}{0.59} = 3.13\,\text{mF} = 3130\,\mu\text{F}.$$