Models

Most electronic devices and circuits are complicated. To get to the main ideas, we often idealize; this means stripping away all unnecessary details. These idealized circuits "model" or represent the behavior of real devices.

The models should be accurate enough to represent the essential features of device and circuit performance, yet simple enough to permit rapid analysis and to enhance our understanding of system performance. If the model is too simple, it will fail to portray essential issues. If the model is too detailed, unnecessarily calculations may obscure understanding of the essential issues.

A model is a group of connected "circuit elements" that can replace the modeled device (or circuit). Circuit elements are described by linear and simpler relation between voltage and current such are resistance, capacitance, inductance and ideal sources.

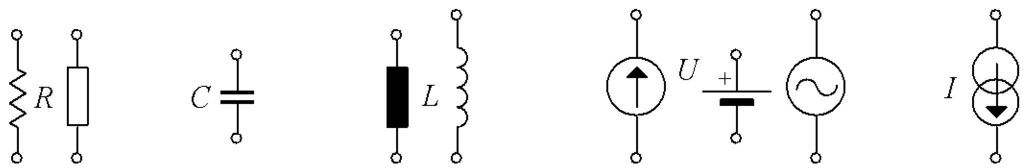


Figure – *Symbols utilized in the electrical schematics for: resistors, capacitors, inductors, the ideal voltage and current sources.*

Resistance

The ideal resistor is a linear element characterized by Ohm's law:

$$v = i \cdot R \text{ (or } i = v \cdot G\text{)}.$$

R is the resistance of the element and G is the conductance.

The plot of v-i characteristic for the ideal resistor is a strait line through the origin with the slope $1/R$.The resistance is a dissipative element, it dissipate a power:

$$P = v \cdot i = (i \cdot R) \cdot i = i^2 R \left(= v \cdot \frac{v}{R} = \frac{v^2}{R} \right)$$

Ideal independent sources

An ideal "voltage source" is an element that fixes the voltage between its terminals regardless of the size or direction of the current flowing through the source. The current drawn from the source depends on the external circuitry (for a resistor R connected at the source terminals the current can be computed by Ohm's law: $i = v/R$).

An ideal "current source" is an element that maintains a fixed current regardless of the voltage between its terminals. The voltage between the source terminals depends on the external circuitry (for a resistor R connected at the source terminals the voltage can be computed by Ohm's law: $v = i \cdot R$).

Basic network configurations – Linear resistive networks

Resistors in series

We will analyze a voltage source in series with two resistors (schematics in the next figure).

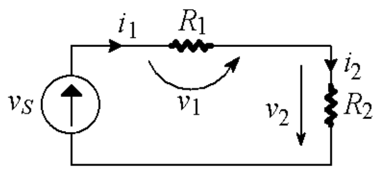


Figure – *Voltage divider;*
The output voltage v_2 is a part of the source voltage v_S, that depends on the resistors ratio.

The circuit will be analyzed with the Kirchhoff and Ohm's laws. The Kirchhoff current law (KCL) gives same current in a circuit without ramifications: $i_1 = i_2 = i$.
The Kirchhoff voltage law (KVL) can be expressed in the "gravitational" form (from top to bottom in a schematic); in our circuit the source voltage is over the two resistors:

$$v_S = v_1 + v_2 = R_1 i_1 + R_2 i_2 = (R_1 + R_2) i \quad \Rightarrow \quad i \left(= i_1 = i_2\right) = \frac{v_S}{R_1 + R_2}$$

The voltage drop over each resistance was replaced according to Ohm's law and the current in the circuit was computed. The current is the same that the current in a single resistor with a value $R_1 + R_2$. A first conclusion is that the **resistances in series add**.
Once the current flowing in the resistances is known, the voltage across each resistance can be found from Ohm's law:

$$v_1 = R_1 i = \frac{R_1}{R_1 + R_2} v_S , \qquad v_2 = \frac{R_2}{R_1 + R_2} v_S .$$

A second conclusion is that the voltage across resistors in series gets divided proportional to the resistance of each element. The circuit analyzed is called **voltage divider** and the relation for computing the voltage at the divider output, v_2, is called the voltage divider rule.

Resistors in Parallel

The circuit in the next figure is a current source connected in parallel with two resistances.

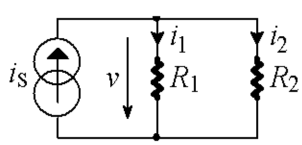


Figure – *Current divider;*
The output current v_2 is a part of the source current i_S, that depends on the resistors ratio.

Since the tree elements are in parallel, they must all have the same voltage across them.
The Kirchhoff current law combined with Ohm's law applied to both resistances gives:

$$i_S = i_1 + i_2 = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{R_1 + R_2}{R_1 R_2} v = \frac{v}{R_{ep}} ,$$

The first conclusion is that total voltage across a parallel combination of R_1 and R_2 is the same as that which occurs across a single resistance of value R_{ep}:

$$\frac{1}{R_{ep}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \text{or} \qquad R_{ep} = \frac{R_1 R_2}{R_1 + R_2} .$$

The currents in the resistors are:

$$i_1 = \frac{v}{R_1} = \frac{R_2}{R_1 + R_2} i_S , \qquad i_2 = \frac{R_1}{R_1 + R_2} i_S .$$

The previous formulas indicate how the current from the source is divided between two conducting path. For $R_1 = R_2$ the currents are equal: $i_1 = i_2 = i_S/2$; if R_1 is larger than R_2 more of the current flows in R_2.

This configuration of circuit is called a **current divider** and the relation between the output current i_2 and the input current i_S represents the current divider rule.

Superposition of Independent Sources

A way of breaking up problems containing more independent sources into several smaller problems is called "superposition". Superposition means to determine the response of each independent source, one at a time, assuming that all other independent sources are set to zero (suppressed) and then sum the results to get the total response.
The superposition theorem is a consequence of the linearity of Kirchhoff laws applied to linear circuits. **Superposition can be used ONLY in LINEAR networks!**

Equivalent Circuits

To a terminal pair of a linear network it is an unknown element, used to represent the fact that we might connect many different things there. In many cases the only feature of interest is the relation between the voltage across the terminal pair and the current that flows through these terminals. The relation between terminal voltage and terminal current is called a "terminal characteristic".

Active Network Models – Thevenin and Norton Equivalent Circuits

All sources of electrical energy can be represented in terms of either voltage or current sources. Practical sources may be modeled by a combination of an ideal source and one or more passive circuit components. At relatively low frequencies, the passive component is resistance. The two form of circuit models are:

- the Thévenin model – an ideal voltage source v_S in series with a resistance R_S,
- the Norton model – an ideal current source i_S in parallel with a resistance R_S.

In order to derive these models one can consider a linear resistive network, considered at its terminals (as in the next figure). The network is linear and hence it is characterized by a linear function at its terminals: $v = a \cdot i + b$.

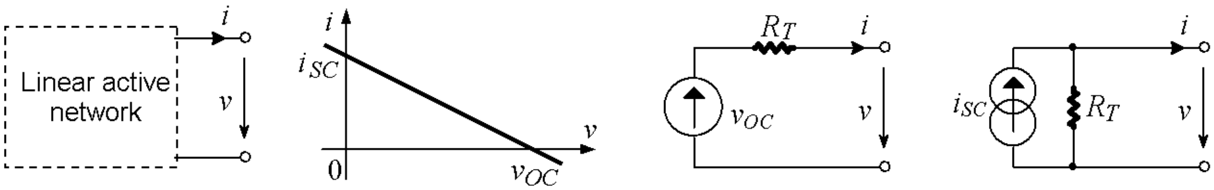


Figure – *Thévenin and Norton equivalent circuits for linear active networks*

The Thévenin model is more convenient when the internal resistance of the source is very small compared to external load resistance to be connected and the Norton model is more convenient when the internal resistance is very large compared to external load resistance. An ideal voltage source is said to have zero internal resistance and the ideal current source is said to have infinite internal resistance.
The source models are considered independent sources, since the value of v_S or i_S does not depend on some other circuit variable.

Controlled source models

For an ideal amplifier the output should be a constant times the input. The input and the output of an amplifier may be either voltage or current. Thus, there are four possible combinations of input-output control:

1. Voltage-controlled voltage source
2. Voltage-controlled current source
3. Current-controlled voltage source
4. Current-controlled current source.

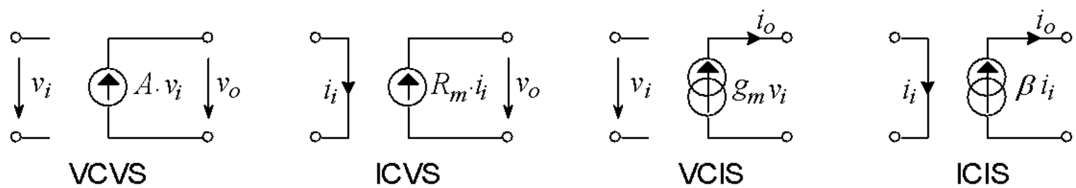


Figure – *Four possible models of ideal controlled sources in electronic circuits*

Voltage-controlled voltage source

The most common combination is a voltage-controlled voltage source (VCVS). The output voltage is:

$$v_o = Av_i$$

The quantity *A* is the voltage gain, and it is dimensionless. This model could be used to represent the voltage amplifier.

The VCIS output is:

$$i_o = g_m v_i$$

The constant g_m is the transconductance of the device and it has the units of siemens (S).

The value of the voltage at the ICVS is:

$$v_o = R_m i_i$$

The constant R_m is the transresistance of the device and it has units of ohms (Ω).

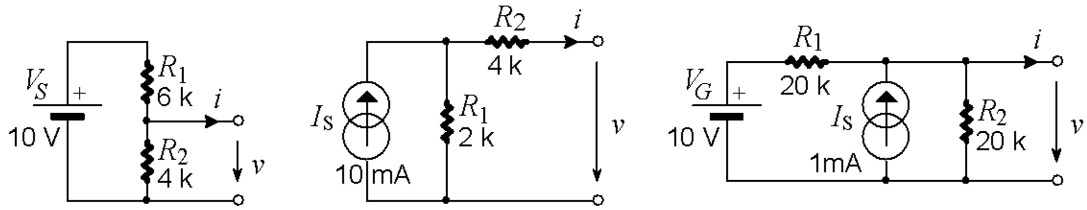
The value of the output current at the ICIS is:

$$i_o = \beta i_i$$

The constant β is the current gain and it is dimensionless. Bipolar junction transistors (BJT) in their ideal form may be represented as ICIS. Field effect transistors (FET) may be represented by the VCIS model.

Applications

Find the Thevenin and Norton equivalent circuit of the network at the indicated terminal pair:



P1. *P2.* *P3.*

For the 3rd circuit (*P3*):

I. compute using KVL, KCL and Ohm's law;
II. compute using the superposition theorem.

P4. Find out the parameters of the current generator that gives 0.91 mA current through a 5 kΩ resistance and 0.98 mA current through a 1 kΩ resistance.

Solutions

Equivalent sources parameters: $v_{OC} = v\big|_{i=0}$, $i_{SC} = i\big|_{v=0}$, $R_T = \dfrac{v_{OC}}{i_{SC}}$ or $R_T = \dfrac{v}{-i}\Big|_{V_S, I_S=0}$.

Notations: i_1, v_1 – current through R_1, voltage across R_1 and i_2, v_2 – R_2 current, voltage.

P1.

1st goal – compute Thevenin (open-circuit) voltage: V_T (that is $v_{OC}=v$ for $i=0$).

Directly – voltage divider rule: $V_T = v_{OC} = \dfrac{R_2}{R_1+R_2} V_S = \dfrac{4k}{6k+4k} 10 = 4\,\text{V}$.

Detailed computing:

KVL: $V_S = v_1 + v_2$

Ohm's laws: $v_1 = R_1 \cdot i_1$ $\quad v_2 = R_2 \cdot i_2$

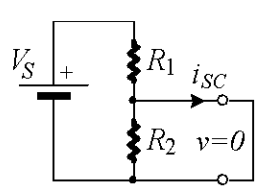
Replace in KVL: $V_S = R_1 \cdot i_1 + R_2 \cdot i_2$

KCL for output node: $i_1 = i_2 + i$, $i = 0$ => $i_1 = i_2 = \dfrac{V_S}{R_1+R_2}$

Thevenin (open-circuit) voltage: $V_T = v_{OC} = v_2 = R_2 \cdot \dfrac{V_S}{R_1+R_2} = \dfrac{R_2}{R_1+R_2} V_S$.

2nd goal – compute Norton (short-circuit) current (that is $i_{SC}=i$ for $v=0$).

The circuit with short-circuit at the output:



Directly – current in R_1 (R_2 voltage/current being zero): $I_N = i_{SC} = \dfrac{V_S}{R_1} = \dfrac{10}{6k} = 1.67\,\text{mA}$.

Detailed computing:

Ohm's law for R_2: $i_2 = \frac{v_2}{R_2} = \frac{v}{R_2} = 0$

KCL for output node: $i_1 = i_2 + i_{SC} = i_{SC}$

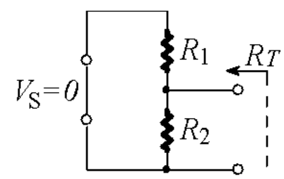
KVL: $V_S = v_1 + v_2 = v_1 + v = v_1$

Ohm's law for R_1: $i_1 = \frac{v_1}{R_1} = i_{SC} = \frac{V_S}{R_1}$.

3[rd] goal – compute Thevenin/ Norton resistance.

1. Based on v_{OC} and i_{SC}: $R_T = \frac{v_{OC}}{i_{SC}} = \frac{4}{1.67m} = 2.4\,\text{k}\Omega$.

2. The resistance seen from the output to the circuit, based on the circuit with the independent source suppressed ($V_S=0$); One can see that the resistance seen from the output consists of the two resistances (R_1 and R_2) in parallel:



$$R_T = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{6k \cdot 4k}{6k + 4k} = \frac{24k}{10} = 2.4\,\text{k}\Omega;$$

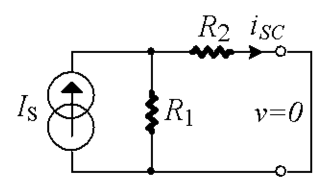
the same result that was obtained with the previous method. In most of the practical cases there is necessary only one equivalent circuit, e.g. the Thevenin equivalent, and it is enough to compute v_{OC} and R_T; in this case the second method for computing R_T is simpler (it is not necessary to compute i_{SC}).

P2.

1[st] goal – compute Norton (short -circuit) current: I_N (that is $i_{SC}=i$ for $v=0$).

Directly – current divider rule:

$$I_N = i_{SC} = \frac{R_1}{R_1 + R_2} I_S = \frac{2k}{2k + 4k} 10m = 3.33\,\text{mA}.$$



Detailed computing:

KCL: $I_S = i_1 + i_2$

Ohm's laws: $i_1 = \frac{v_1}{R_1}$ $\quad i_2 = \frac{v_2}{R_2}$

KVL: $v_1 = v_2$

Replace in KCL: $I_S = \frac{v_1}{R_1} + \frac{v_2}{R_2} = v_2 \frac{R_1 + R_2}{R_1 R_2}$, $\quad v_2 = \frac{R_1 R_2}{R_1 + R_2} I_S$

Ohm's laws for R_2: $i_{SC} = i_2 = \frac{v_2}{R_2} = \frac{1}{R_2} \frac{R_1 R_2}{R_1 + R_2} I_S = \frac{R_1}{R_1 + R_2} I_S$.

2[nd] goal – compute Thevenin (open -circuit) voltage (that is $v_{OC}=v$ for $i=0$).

Directly – voltage over R_1 (R_2 voltage/current being zero):

$$V_T = v_{OC} = R_1 I_S - R_2 \cdot i = 2k \cdot 10m - 0 = 20\,\text{V}.$$

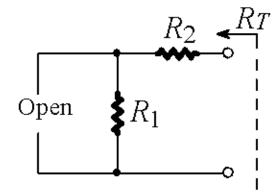
Detailed computing:

Ohm's law for R_2: $v_2 = R_2 \cdot i_2 = R_2 \cdot i = 0$

KCL: $I_S = i_1 + i_2 = i_1 + i = i_1$

Ohm's law for R_1: $v_1 = R_1 \cdot i_1 = R_1 I_S$.

KVL: $v_1 = v_2 + v = 0 + v = v_{OC} = R_1 I_S$



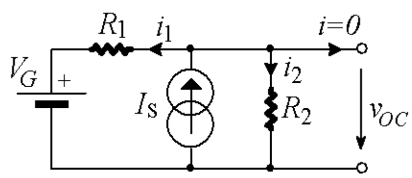
3[rd] goal – compute Thevenin/ Norton resistance.

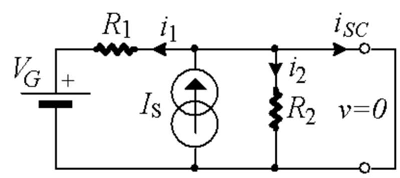
1. Based on v_{OC} and i_{SC}: $R_T = \dfrac{v_{OC}}{i_{SC}} = \dfrac{20}{3.33m} = 6\,\text{k}\Omega.$

2. The resistance seen from the output to the circuit, based on the circuit with the independent source suppressed (I_S=0);

One can see that the resistance seen from the output consists of the two resistances (R_1 and R_2) in series: $R_T = R_1 + R_2 = 2k + 4k = 6\,\text{k}\Omega$; the same result that was obtained with the previous method.

P3.I. We will use the currents direction through the resistors as in the next figures; the other direction can be also selected and the current signs will reverse.





1[st] goal – compute Thevenin (open-circuit) voltage: V_T (that is $v_{OC}=v$ for $i=0$), on the upper left schematics.

KCL: $I_S = i_1 + i_2, \quad i_1 = I_S - i_2,$

KVL + Ohm's law: $V_G = -R_1 i_1 + R_2 i_2, \quad V_G = -R_1 I_S + R_1 i_2 + R_2 i_2, \quad i_2 = \dfrac{V_G + R_1 I_S}{R_1 + R_2}$

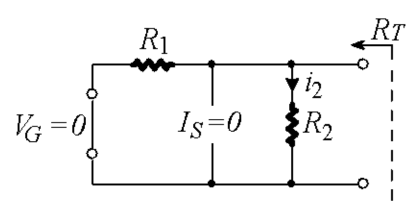
Ohm's law on R_2: $V_T = v_{OC} = R_2 i_2 = \dfrac{R_2}{R_1 + R_2}\left(V_G + R_1 I_S\right) = \dfrac{1}{2}\left(10 + 1m \cdot 20k\right) = 15\,\text{V}.$

2[nd] goal – compute Norton (short-circuit) current (that is $i_{SC}=i$ for $v=0$), on the upper-right schematics.

KCL: $I_S = i_1 + i_2 + i_{SC} = i_1 + i_{SC}, \quad i_{SC} = I_S - i_1,$

KVL + Ohm's law: $V_G = -R_1 i_1 + v = -R_1 i_1, \quad i_1 = -\dfrac{V_G}{R_1}$

Replace i_1: $i_{SC} = I_S + \dfrac{V_G}{R_1} = 1m + \dfrac{10}{20k} = 1.5\,\text{mA}.$



3[rd] goal – compute Thevenin/ Norton resistance.

1. Based on v_{OC} and i_{SC}: $R_T = \dfrac{v_{OC}}{i_{SC}} = \dfrac{15}{1.5m} = 10\,\text{k}\Omega.$

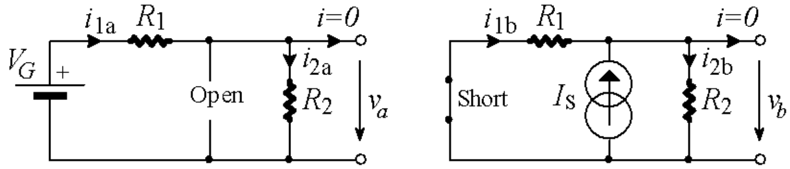
2. The resistance seen from the output to the circuit, based on the circuit with the independent source suppressed (V_G=0, I_S=0);

One can see that the resistance seen from the output consists of the two resistances (R_1 and R_2) in parallel: $R_T = R_1 \parallel R_2 = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{20k \cdot 20k}{20k + 20k} = 10\,\text{k}\Omega$;

the same result that was obtained with the previous method.

P3.II. Compute with superposition theorem.

1st goal – Thevenin (open-circuit) voltage: V_T (that is $v_{OC}=v$ for $i=0$): $V_T = v_a + v_b$.

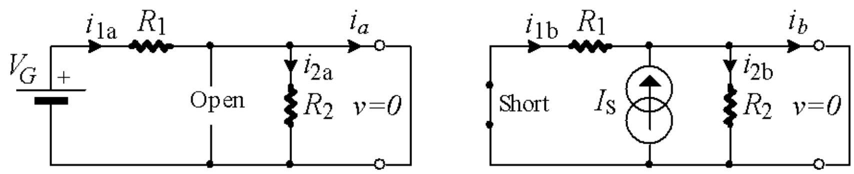


a) Voltage divider rule (on upper left schematics): $v_a = \dfrac{R_2}{R_1+R_2} V_G$;

b) Voltage over the equivalent parallel resistance: $v_b = I_S \dfrac{R_1 R_2}{R_1+R_2}$.

Superposition: $V_T = \dfrac{R_2}{R_1+R_2} V_G + I_S \dfrac{R_1 R_2}{R_1+R_2} = \dfrac{R_2}{R_1+R_2}\left(V_G + I_S R_1\right) = 15\,\mathrm{V}$.

2nd goal –Norton (short-circuit) current (that is $i_{SC}=i$ for $v=0$): $I_N = i_a + i_b$



a) Ohm's law for R_1 (on upper left schematics): $i_a = i_{1a} = \dfrac{V_G}{R_1}$, $\left(i_{2a} = \dfrac{v}{R_2} = 0\right)$;

b) KCL: $I_S = i_b + i_{2b} - i_{2a} = i_b$, $\left(i_{2a} = i_{2b} = \dfrac{v}{R_1} = \dfrac{v}{R_2} = 0\right)$;

Superposition: $I_N = i_a + i_b = \dfrac{V_G}{R_1} + I_S = 1.5\,\mathrm{mA}$.

3rd goal – compute Thevenin/ Norton resistance: as for *P3I*.