Op-amp ac Effects and Limitations

A practical op-amp provides high gain from dc up to a given frequency beyond which gain decreases with frequency and the output is also delayed with respect to the input. This affects both its frequency and transient response.

Open Loop Response

The general behavior of open-loop gain as a function of frequency is presented in the next figure; there is a log-log plot with amplitude in dB, frequency at a logarithmic scale.

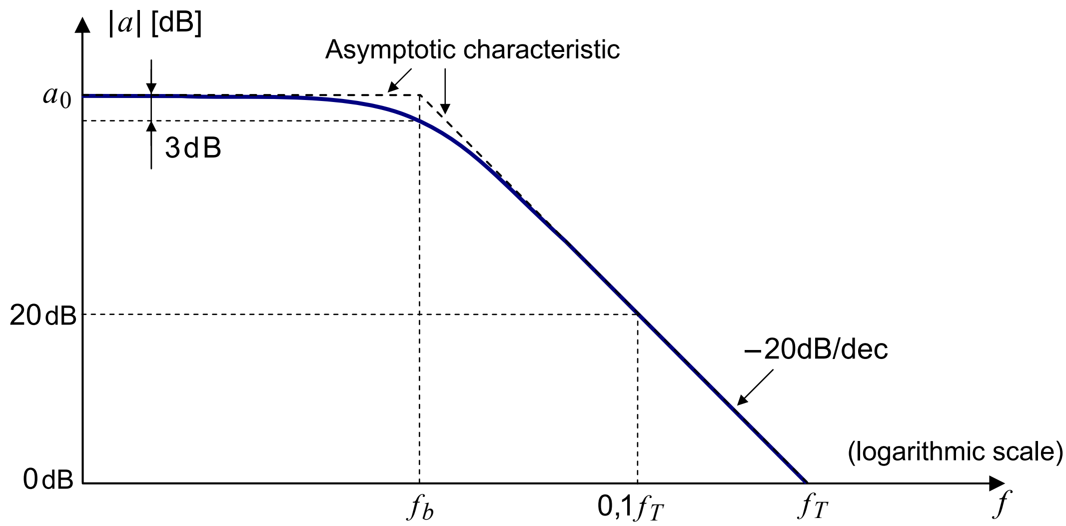

Figure – *Magnitude of open-loop gain for a typical op-amp as a function of frequency*

For stability purposes, the open-loop gain functions of most op-amps are deliberately established at -20 dB/decade by an internal compensation circuit.
Thus, in terms of absolute amplitude, the response on a log-log plot well above f_b, is affected by a factor of 0.1 (drops 10 times) for tenfold increase in frequency.

The transfer characteristic of the internally compensated op-amps is dominated by a single low-frequency pole:

$$a(jf) = \frac{a_0}{1 + j\dfrac{f}{f_b}},$$

where a_o is the open-loop gain at dc and low frequencies and,
f_b is the open-loop break frequency (also called: -3dB frequency or corner frequency).
The magnitude and phase of the transfer function are:

$$|a(f)| = \frac{a_0}{\sqrt{1 + (f/f_b)^2}}, \qquad \varphi(f) == -\arctan\left(\frac{f}{f_b}\right).$$

The frequency at which gain becomes unity is called the unity-gain frequency or transition frequency f_T – it marks the transition from amplification to attenuation. The transition frequency f_T is an op-amp parameter that is given in the op-amp data-sheet.

In the frequency range well above f_b, the gain can be approximated as:

$$|a(f)| = \frac{a_0}{f/f_b} \text{ for } f >> f_b .$$

which provides the -20dB/octave roll-off expected.
The previous formula written for the transition frequency:

$$|a(f_T)| = \frac{a_0}{f_T/f_b} = 1 \text{ gives } f_T = a_0 f_b .$$

Stated in words, the unity-gain frequency is the product of the dc (or low-frequency) gain and the break frequency.
A more general result can be found from the previous formulas:

$$|a(f)| \cdot f = a_0 f_b = f_T$$

It states that the gain-bandwidth product is a constant equal to the unity-gain frequency. Both terms unity-gain frequency and gain-bandwidth product can be used in specifications.

Closed Loop Bandwidth

For the non-inverting amplifier the closed loop gain is:

$$A = \frac{a}{1 + a \cdot b}.$$

By replacing the gain-frequency characteristic of the internally compensated op-amps (dominated by a single low-frequency pole), one can find the transfer characteristic of the closed-loop amplifier as a function of frequency:

$$A(jf) = \frac{a_0}{1 + j\frac{f}{f_b}} \cdot \frac{1}{1 + \frac{a_0 b}{1 + j\frac{f}{f_b}}} = \frac{a_0}{1 + j\frac{f}{f_b} + a_0 b} = \frac{a_0}{1 + a_0 b} \cdot \frac{1}{1 + \frac{jf}{f_b(1 + a_0 b)}} = \frac{A_0}{1 + j\frac{f}{f_B}}$$

where A_o is the closed-loop gain at dc and low frequencies and,
f_B is the closed-loop break frequency (or close-loop -3dB frequency).
The result indicates an extension of closed-loop amplifier bandwidth by a factor equal to the feedback factor ($1+a_0 b$):

$$f_B = f_b(1 + a_0 b) = f_b \frac{a_0}{A_0} = \frac{f_T}{A_0}$$

It can be also stated that the closed-loop bandwidth is the unity-gain frequency over the dc closed-loop gain.

The gain-bandwidth product is the same for the open-loop and the closed-loop in the non-inverting configuration (equal to the op-amp unity-gain frequency):

$$|A(f)| \cdot f = A_0 f_B = f_T .$$

A graphical interpretation of this result (as indicated in the next figure) is that intersection of the horizontal line indicating the closed-loop gain with the open-loop gain plot gives the break frequency of the closed-loop amplifier (if the dc closed-loop gain is significantly lower than the dc open-loop gain).

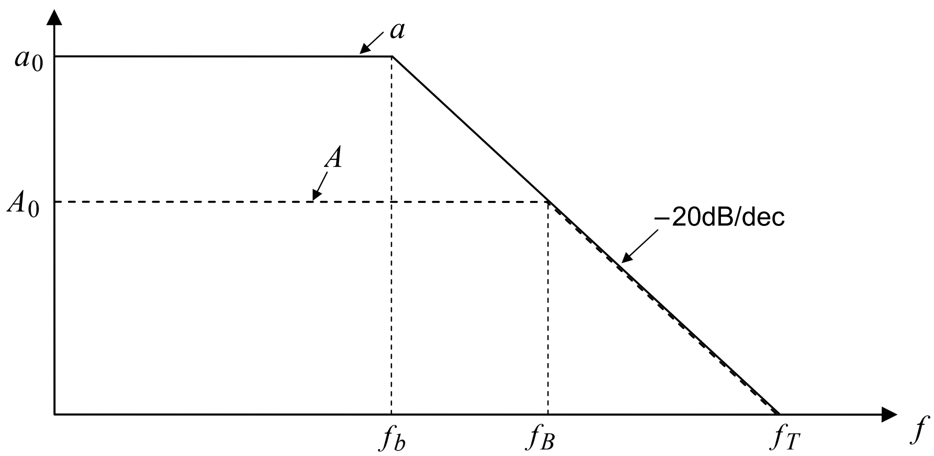

Figure – *Graphical interpretation of closed-loop bandwidth for a non-inverting amplifier*

For the inverting amplifier the magnitude of the (ideal) voltage gain is different than the inverse of the feedback factor:

$$\left|A\right| = \left|-\frac{R_2}{R_1}\right| = \frac{R_2}{R_1} \text{ and } b = \frac{R_1}{R_1 + R_2}.$$

The gain-bandwidth product (GBP) of the inverting configuration will be lower than the one of the non-inverting configuration (f_T):

$$GPB = \left|A_0\right| f_B = \frac{R_2}{R_1} f_T \frac{R_1}{R_1 + R_2} = \frac{R_2}{R_1 + R_2} f_T < f_T$$

Transient Response

The transient response is the response of an amplifier to a step input as a function of time.

The rise time

The voltage follower transfer function is: $A(jf) = \dfrac{1}{1 + j\dfrac{f}{f_T}}$.

It indicates a pole at angular frequency $\omega = 2\pi \cdot f_T$.

As a result the time response of the voltage follower is:

$$v_O(t) = V_I\left[1 - \exp\left(-\frac{t}{\tau}\right)\right] \text{ with } \tau = \frac{1}{2\pi \cdot f_T}.$$

The rising time t_R is taken from v_O to swing from 10% to 90% of its amplitude:

$$t_R = \tau\left(\ln 0.9 - \ln 0.1\right) = \frac{\ln 0.9 - \ln 0.1}{2\pi}\frac{1}{f_T} = \frac{0.35}{f_T}.$$

This relationship represents a link between the frequency domain parameter f_T and the time domain parameter t_R; the higher f_T, the lower t_R.

The concept of rise time applies equally well to fall time. Thus if it takes a certain amount of time for a pulse to reach a final value at the output of an amplifier, it will take a comparable amount of time for the output to return to zero when the pulse is removed from the input.

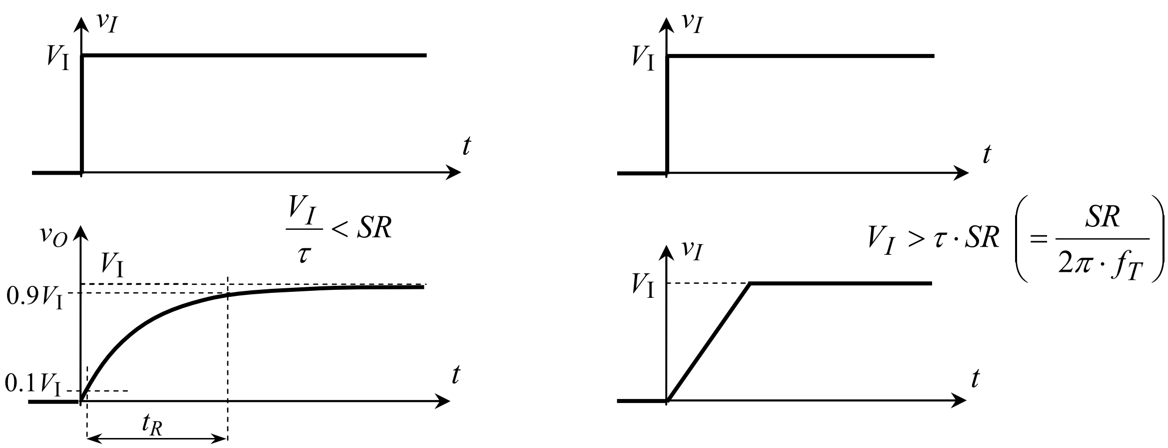

Figure – *Step function input and the corresponding output of the voltage follower*

Slew Rate

In practice it is observed that above certain step amplitude the output slope saturates at a constant value called the slew rate (SR). Slew rate limiting comes from the limited ability of the internal circuitry to charge or discharge the frequency-compensating capacitances. SR is expressed in volts over seconds (V/s). A typical value for the general purpose op-amp LM741 is 0.5 V/μs. Op-amps for faster applications have much greater SR, tens of V/μs or even greater.

Full Power Bandwidth

The effect of slew rate limiting is to distort the output signal whenever an attempt is made to exceed the SR capabilities of the op-amp.
The rate of change of a sine output voltage:

$$v_o = V_{o\_pk} \sin 2\pi \cdot f \cdot t \qquad \text{is:} \qquad \frac{dv_o}{dt} = 2\pi \cdot f \cdot V_{o\_pk} \sin 2\pi \cdot f \cdot t\,.$$

To prevent distortion we must require:

$$\left(\frac{dv_o}{dt} \right)_{Max} \le SR \qquad \text{or} \qquad f \cdot V_{o\_pk} \le \frac{SR}{2\pi}\,.$$

That indicates a tradeoff between frequency and amplitude of the output voltage.
If we want to exploit the full bandwidth f_T of a LM741 voltage follower, for example, then we must keep:

$$V_{o\_pk} \le \frac{SR}{2\pi \cdot f_T} \cong 80\,\text{mV}\,.$$

For an undistorted ac output with $V_{o\_pk}=1$ V, a LM741 follower must be operated below

$$f_{\max} = \frac{SR}{2\pi \cdot V_{o\_pk}} = \frac{0.5}{2\pi \cdot 1}\frac{1}{\mu} \cong 0.08\,\text{MHz} = 80\,\text{KHz}\,.$$

That value is way bellow the op-amp $f_T = 1$ MHz.

The full power bandwidth (FPB) is the maximum frequency at which the op-amp will yield an undistorted ac output with the largest possible amplitude.
Assuming symmetric output saturation values of +/–V_{sat} we get:

$$FPB = \frac{SR}{2\pi \cdot V_{sat}}.$$

For a 741 op-amp with V_{sat} =13 V, the FPB is as low as 6 kHz.
A note can be made regarding the op-amps frequency limitation: For a given peak output voltage, the rise time due to SR is independent of the gain of the amplifier; by contrast, the closed loop gain strongly affects the bandwidth and the corresponding rise time.

Combination of Linear Bandwidth and Slew Rate

In the previous analysis the effects of open-loop bandwidth and slew rate were considered independently; in practice both finite bandwidth and slew rate limitations are always present.
From the signal point of view it is convenient to consider either complex analog signals or pulse type signals. The given application will determine which form is better to be used.

Analog complex signal

To avoid distortion of an analog signal amplified by an op-amp, the conditions are:

1. $f_B >> f_H$ where $f_B = b \cdot f_T$ is the closed-loop bandwidth,
2. $f_{SR} > f_H$ where $f_{SR} = \frac{SR}{2\pi \cdot V_{o\_pk}}$ is the frequency limited by SR,

and f_H represents the highest frequency contained in the input analog signal.
For a precision amplifier it should be used an op-amp with a wider bandwidth (higher f_T) and a design with low gain in the given stage (higher b=1/A_0). The increase of inequality decreases the error in gain due to finite bandwidth effect (e.g. for $f_B = 10\,f_H$, the gain error is very low, approx. 0.05 dB).
The slew rate should be at least large enough to track the output signal at the highest frequency/amplitude contained in the signal (SR limitations depends on the output signal level, but is independent on the gain).

Pulse type of signals

If the op-amp is used in some digital or pulse applications it is necessary that:

1. $t_R << t_i$ where $t_R = \frac{0.35}{f_T}$ is the rise time,
2. $t_{SR} < t_i$ where $t_{SR} = \frac{V_o}{SR}$ is the total time for the output signal to change,

and t_i represents the rise time at the output.
The actual rise time at the output can be approximated as:

$$t_o = \sqrt{t_R^2 + t_{SR}^2}.$$

From this formula it can be observed that for: $t_R < 0.1\ t_i$, the percentage increase in output time can be neglected (it is less than 0.5%).