Linear Op Amp Circuits

Circuits presented here have frequency-dependence properties incorporated in the design. Such circuits usually employ capacitors: differentiator, integrator, all phase shift circuits and op-amp amplifiers with single power supply.

Integrator Circuits

RC Integrator

The simpler integrator circuit is the low-pass RC network presented in next figure; the circuit (*a*) and the operational form (*b*).

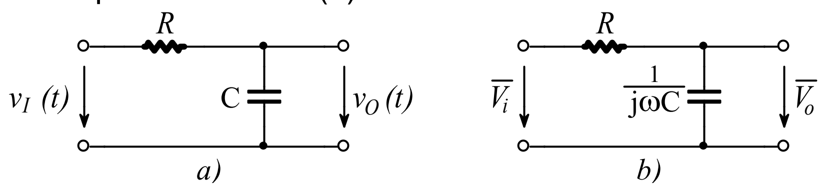

Figure – the *RC integrator (RC low-pass filter) – a) actual circuit, b) operational form*

The voltage divider rule gives the transfer function of the circuit:

$$\overline{V}_o = \frac{1}{1 + j\omega \cdot RC} \cdot \overline{V}_i\,, \quad H(j\omega) = \frac{\overline{V}_o}{\overline{V}_i} = \frac{1}{1 + j\omega \cdot RC}$$

Since $H(j\omega)$ is a complex function, it can be expressed in polar form. The amplitude (or magnitude) response $M(\omega) = |H(j\omega)|$ is the amplitude of the complex function and the phase response: $\theta(\omega)$ is the phase shift of the complex function. The amplitude response is usually expressed in decibels (dB): $M_{dB}(\omega) = 20 \cdot \lg M(\omega)$.

Based on this example it will be recalled the circuit analysis method known as Bode plot (that method permits determination of amplitude and phase responses for rather complex circuits by some simplified graphical procedures).

Amplitude and phase response

The point where $\omega RC = 1$ corresponds to the point where $M_{db}(\omega) = -3$ dB. This frequency is called the break frequency or corner frequency:

$$\omega_b = \frac{1}{RC}\,, \quad f_b = \frac{1}{2\pi RC}$$

The transfer function can be rewritten:

$$H(j\omega) = \frac{1}{1 + j\dfrac{\omega}{\omega_b}}\,, \text{ or } H(j\omega) = \frac{1}{1 + j\dfrac{f}{f_b}}\,.$$

The amplitude and phase response (numerator minus denominator angles) are:

$$M(\omega) = \frac{1}{\sqrt{1 + (\omega/\omega_b)^2}} \text{ and } \theta(\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_b}\right).$$

These results can be simplified in different frequency domains as follows:

for $\omega << \omega_b$: $M(\omega) = 1$ (= 0 dB), $\theta(\omega) = 0$;

for $\omega >> \omega_b$: $M(\omega) = \frac{\omega_b}{\omega}$ (= −20 dB / decade), $\theta(\omega)$ = −90 degree, and

for $\omega = \omega_b$: $M(\omega) = \frac{1}{\sqrt{2}}$ (= −3 dB), $\theta(\omega)$ = −45 degree;

The first two cases give asymptotic lines used in the simplified Bode plot.

With Bode plot analysis the actual curve is often approximated by the break-point approximation. With this simplified curve, the amplitude response is assumed to be constant at 0-dB level at all frequencies in the range $f < f_b$. At $f = f_b$ the curve "breaks" at a slope of – 20 dB/decade.

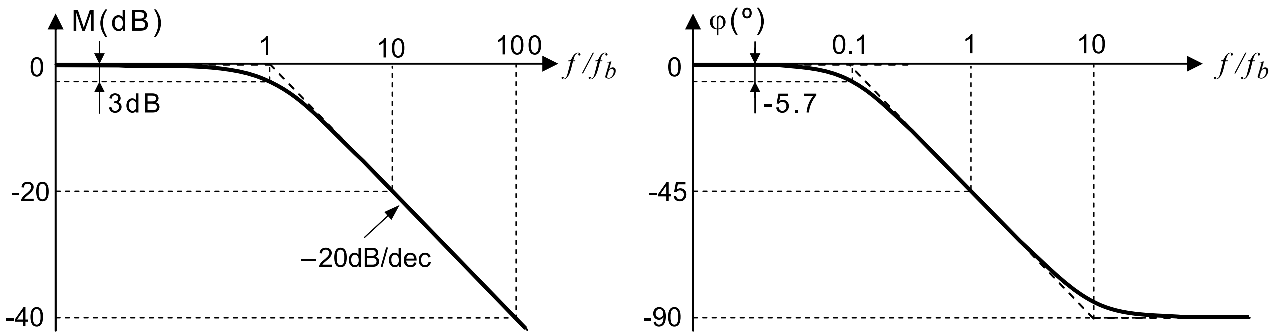

Figure – *the Bode plot for the RC low-pass filter (magnitude, phase response)*

Such a simple circuit is used as a rectifier filter. If the time constant of this circuit (RC product) is much greater than the period of the input signal, the output voltage is approximately constant at the dc (or medium) value of the input voltage.

True Integrator Circuit

The output voltage v_O for an ideal integrator circuit is:

$$v_O(t) = \int_0^t v_I(t)\,dt + v_O(0)$$

The voltage across a capacitor is proportional to the time integral of the capacitor current:

$$v_C(t) = \frac{1}{C}\int_0^t i_C(t)\,dt + v_C(0).$$

If the capacitor is placed as a feedback element in an op–amp circuit as shown in the next figure, the resistance R converts the input voltage in the capacitor current:

$$i_C(t) = \frac{v_I(t)}{R}$$

Since the positive reference terminal of the capacitor voltage v_C is on the left, the output voltage is $v_O(t) = -v_C(t)$ and the net result for the output voltage is then:

$$v_O(t) = \frac{-1}{RC}\int_0^t v_I(t)\,dt + v_O(0)$$

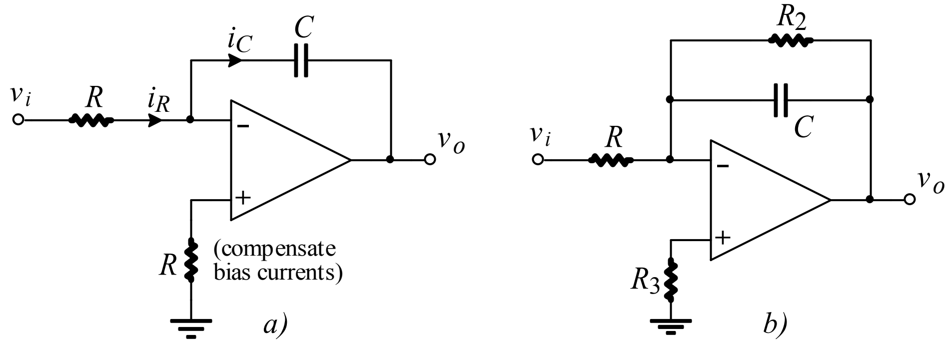

Figure – *The integrator circuit: a) true integrator, b) ac integrator.*

This result can be transformed to the ideal integrator equation with an additional inversion and gain (or by selecting R and C such that RC product to b 1). The circuit that results is called a **true integrator circuit**.
The dc non-idealities of a real op-amp (dc offset voltage and bias currents) are equivalent to a small dc source at the input. The integration of a (small) dc input voltage will produce a (slow) move of the output towards saturation.

ac Integrator Circuit

The true integrator does not operate satisfactory with general purpose op-amp due to the integration effect of the dc offset voltage and bias currents.
This problem can be investigated utilizing the frequency response function of the integrator.
The transfer function of the true integrator is:

$$H(j\omega) = \frac{-\overline{Z}_2}{\overline{Z}_1} = \frac{-1/j\omega \cdot C}{R} = -\frac{1}{j\omega \cdot RC}.$$

The gain magnitude is very large at low frequencies:

$$M(\omega) = |H(j\omega)| = \frac{1}{\omega RC}.$$

The theoretical value of the amplitude response of the true integrator is infinite at dc.
The gain can be dropped at dc by placing a resistor R_2 in parallel with the capacitor as shown in the next figure. This circuit is the **ac integrator**. Since the capacitor is an open circuit at dc, the circuit reduces to a simple inverting amplifier with the gain $-R_2/R$ at dc.
The operation of this circuit should eventually approach that of a true integrator as the frequency increases.
The frequency response of an ac integrator can be found with the impedance
$\overline{Z}_2 = R_2 \parallel \frac{1}{j\omega \cdot C} = \frac{R_2/j\omega \cdot C}{R_2 + 1/j\omega \cdot C} = \frac{R_2}{1 + j\omega \cdot R_2 C}$ and it is an one pole low-pass filter response:

$$H(j\omega) = \frac{-\overline{Z}_2}{\overline{Z}_1} = \frac{-R_2/R}{1 + j\omega \cdot R_2 C} = \frac{-R_2/R}{1 + j\omega\tau_B},$$

where $\tau_B = R_2 C$ is the time constant of the feedback circuit.
The amplitude response corresponding to the transfer function is:

$$M(\omega) = \frac{R_2/R}{\sqrt{1 + (\omega \cdot R_2 C)^2}} = \frac{R_2/R}{\sqrt{1 + (\omega\tau_B)^2}}.$$

The Bode plot approximation of this is shown in the next figure together with the amplitude response of the true integrator.

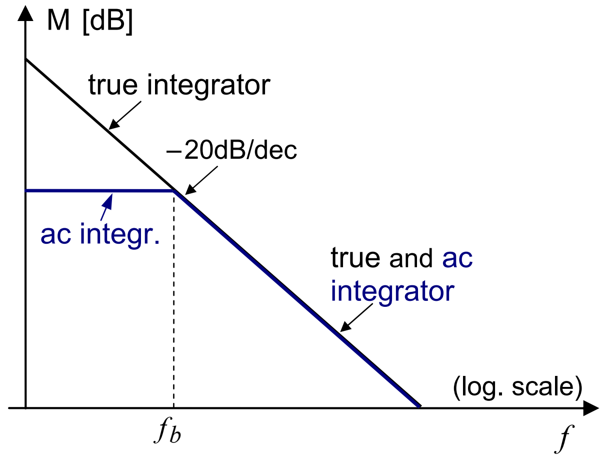

Figure – *Bode break-point approx. for amplitude response of true and ac integrators*

The break frequency corresponding to time constant of the feedback circuit is:

$$f_B = \frac{1}{2\pi\tau_B} = \frac{1}{2\pi \cdot R_2 C}.$$

The amplitude response of ac integrator can be simplified in low and high frequency domains:

- for $f << f_B$: $M(\omega) \cong \frac{R_2}{R}$, the circuit is acting as a constant gain amplifier,
- for $f >> f_B$: $M(\omega) \cong \frac{R_2/R}{\omega \cdot R_2 C} = \frac{1}{\omega \cdot RC}$, the circuit is acting as a true integrator.

The ac integrator circuit performs the signal-processing operation:

$$v_O(t) = -\frac{1}{RC}\int_0^t v_I(t)\,dt.$$

Waveshaping Applications

One useful application of the integrator circuit is in waveshaping circuits for periodic waveforms. For example, square wave can be converted to a triangular waveform. Assume the input signal is periodic and it can be represented as:

$$v_I(t) = v_i(t) + V_I,$$

where $v_i(t)$ is the ac (time-varying) portion of the input and V_I is the dc value of the input. Assuming that all frequency components of $v_i(t)$ are well above f_B, the ac portion of signal is integrated according to the integrator signal-processing operation. The dc component V_I is simply multiply by the gain constant $-R_2/R$. The output voltage can be expressed as:

$$v_O(t) = -\frac{1}{RC}\int_0^t v_i(t)\,dt - \frac{R_2}{R}V_I.$$

Some care must be exercised with this circuit when a dc component is present in order to remain in the linear region (the dc output level plus the peak level of the output time-varying component should not reach saturation).

Differentiator Circuits

RC Differentiator

The simpler differentiator circuit is a high-pass RC network (as the one presented in the next figure, time response of RC differentiator…).

True Differentiator

The output voltage v_O for an ideal differentiator circuit is:

$$v_O(t) = \frac{dv_I(t)}{dt}.$$

The current flow into the capacitor is proportional to the derivative of the capacitor voltage:

$$i_C(t) = C\frac{dv_C(t)}{dt}.$$

The op-amp differentiator circuit is presented in the next figure.
The capacitor is placed as an input element, and since the inverting terminal is virtually grounded, $v_C = v_I$ and the capacitive current is:

$$i_C(t) = C\frac{dv_I(t)}{dt}.$$

The current flows through R and the output voltage is:

$$v_O(t) = -Ri_C(t) = -RC\frac{dv_I(t)}{dt}.$$

This result can be transformed to the ideal differentiator equation with an additional inversion and gain (or by selecting R and C such that RC product to b 1). The circuit that results is called a **true differentiator circuit**.
The noise, which is always present in the electronic circuits, is accentuated strongly by the differentiation process. Noise tends to have abrupt changes, called spikes. Since the output of a true differentiator is proportional to the rate of change of the input, these sudden changes in noise results in pronounced output noise. This problem can be solved by the low-frequency differentiator.

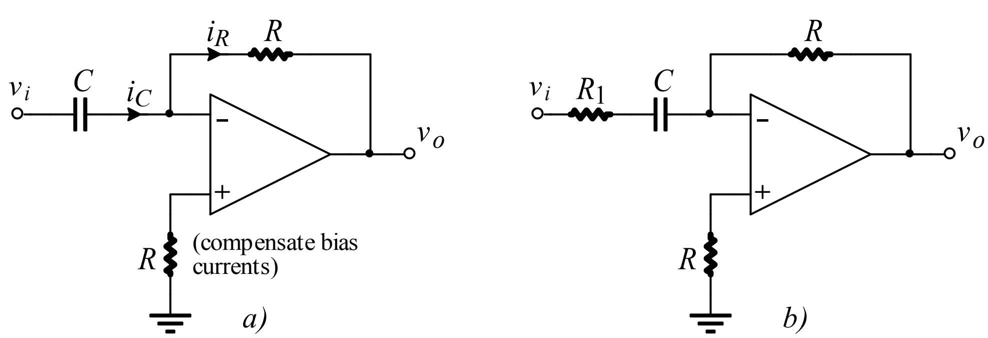

Figure – *The differentiator circuit: a) true differentiator, b) low-frequency differentiator.*

Low-Frequency Differentiator

The noise problem can be investigated utilizing the frequency response function of the true differentiator:

$$H(j\omega) = \frac{-\bar{Z}_2}{\bar{Z}_1} = \frac{-R}{1/j\omega \cdot C} = -j\omega \cdot RC\,.$$

The gain magnitude increase with frequency and it is very large at high frequencies:

$$M(\omega) = |H(j\omega)| = \omega RC\,.$$

The gain can be dropped at high frequencies by placing a resistor R_1 in series with the capacitor as shown in the next figure. This circuit is the **low-frequency differentiator**. Since the capacitor is a short circuit at very high frequencies, the circuit reduces to a simple inverting amplifier with the gain $-R/R_1$ (at very high frequencies). The operation of this circuit should eventually approach that of a true differentiator as the frequency decreases.

The input impedance is: $\bar{Z}_1 = R_1 + \frac{1}{j\omega \cdot C} = \frac{1 + j\omega \cdot R_1 C}{j\omega \cdot C}$ and the transfer function of the low-frequency differentiator is a one-pole high-pass form:

$$H(j\omega) = \frac{-\bar{Z}_2}{\bar{Z}_1} = \frac{-j\omega \cdot RC}{1 + j\omega \cdot R_1 C} = \frac{-j\omega \cdot RC}{1 + j\omega\tau_i}\,,$$

where $\tau_i = R_1 C$ is the time constant of the input circuit.

The amplitude response corresponding to this transfer function is:

$$M(\omega) = \frac{\omega \cdot RC}{\sqrt{1 + (\omega \cdot R_1 C)^2}} = \frac{\omega \cdot RC}{\sqrt{1 + (\omega\tau_i)^2}}\,.$$

The Bode plot approximation of this is shown in the next figure together with the amplitude response of the true differentiator.

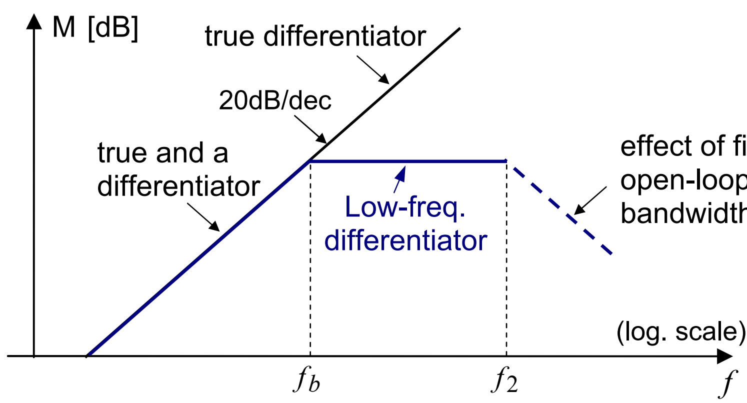

Figure – *Bode break-point approx. for amplitude response of true and ac integrators*

The break frequency corresponding to time constant of the input circuit is:

$$f_b = \frac{1}{2\pi \cdot \tau_i} = \frac{1}{2\pi \cdot R_1 C}\,.$$

The amplitude response of low-frequency differentiator can be simplified in low and high frequency domains:

- for $f >> f_b$: $M(\omega) \cong \frac{R}{R_1}$, the circuit is acting as a constant gain amplifier,
- for $f << f_b$: $M(\omega) \cong \omega RC$, the circuit is acting as a true differentiator.

When the finite open-loop bandwidth of the op-amp is considered, an additional alteration in frequency response occurs illustrated by the dashed line in the previous figure. The frequency introduced by the finite bandwidth f_T (transition frequency) is:

$$f_2 = \frac{f_T}{1 + R/R_1}.$$

If $f << f_b$ the low-frequency differentiator circuit performs the signal-processing operation:

$$v_O(t) = -RC\frac{dv_I(t)}{dt}.$$

Waveshaping Applications

The low-frequency differentiator circuit can be used in waveshaping circuits for periodic waveforms. For example a triangular waveform can be converted to a square wave and a square wave can be converted to a periodic train of narrow “spikes”.