General amplifier concepts

One of the most important applications in the field of analog electronics is the process of amplification. The primary objective of this introductory lecture is to present some of the most general properties of amplifier circuits as well as their circuit model. The intent is to look at the amplifier as a complete system and focus on input, output and gain characteristics. Such properties apply to virtually all types of amplifiers.

Ideal amplifier

An ideal amplifier is characterized by the fact that the output signal is directly proportional to the input signal. Amplifiers in system are often represented by a block diagram; for a voltage amplifier the input voltage signal is denoted as $v_i(t)$ and the output voltage signal is denoted as $v_o(t)$. The quantity t represents time. The functional notation will be replaced by the simplified notation: v_i and v_o.

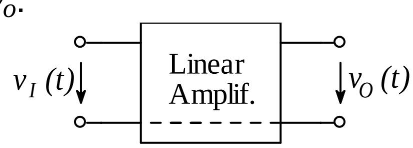

Figure - *Block diagram representation of a linear amplifier*

In this case the quantity A represents the voltage gain. For an ideal amplifier, it can be defined as:

$$A = \frac{v_o(t)}{v_i(t)}$$

It should be stressed that virtually all amplifiers require a dc power supply in order to provide amplification. It is customary to show only signal levels on many block diagrams for signal processing analysis, and the dc power supplies (or bias supplies) are understood to be present. Most active amplifier devices permit a small signal input to control a larger signal output, but the extra power is furnished by the dc power supply.

Source models

All sources of electrical energy can be represented in terms of either voltage or current sources. Practical sources may be modeled by a combination of an ideal source and one or more passive circuit components. At relatively low frequencies, the passive component is resistance. The two form of circuit models are:

- the Thevenin model – an ideal voltage source v_S in series with a resistance R_S,
- the Norton model – an ideal current source i_S in parallel with a resistance R_S.

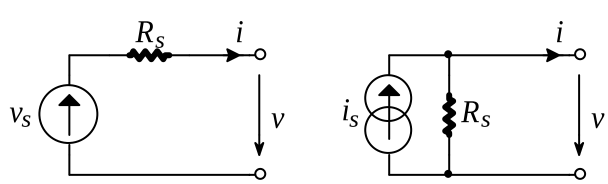

Figure – *Thevenin and Norton equivalent circuits for practical sources*

The Thevenin model is more convenient when the internal resistance of the source is very small compared to external load resistance to be connected and the Norton model is more convenient when the internal resistance is very large compared to external load resistance. An ideal voltage source is said to have zero internal resistance and the ideal current source is said to have infinite internal resistance.
The source models are considered independent sources, since the value of v_s or i_s does not depend on some other circuit variable.

Controlled source models

For an ideal amplifier the output should be a constant times the input. The input and the output of an amplifier may be either voltage or current. Thus, there are four possible combinations of input-output control:

1. Voltage-controlled voltage source (VCVS),
2. Voltage-controlled current source (VCIS),
3. Current-controlled voltage source (ICVS),
4. Current-controlled current source (ICIS).

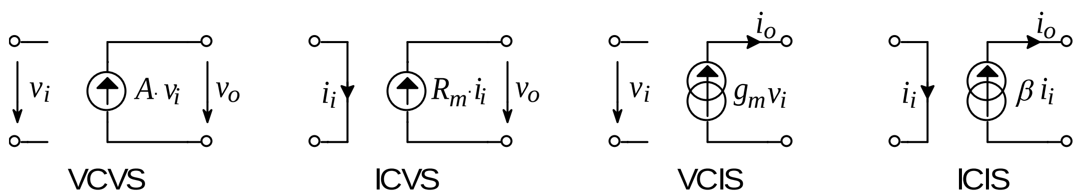

Figure – *Four possible models of ideal controlled sources in electronic circuits*

Voltage-controlled voltage source

The most common combination is a voltage-controlled voltage source (VCVS). The output voltage is:

$$v_o = Av_i$$

The quantity A is the voltage gain, and it is dimensionless. This model could be used to represent the voltage amplifier.

The VCIS output is:

$$i_o = g_m v_i$$

The constant g_m is the trans-conductance of the device and it has the units of siemens (S).

The value of the voltage at the ICVS is:

$$v_o = R_m i_i$$

The constant R_m is the trans-resistance of the device and it has units of ohms (Ω).

The value of the output current at the ICIS is:

$$i_o = \beta i_i$$

The constant β is the current gain and it is dimensionless.

Bipolar junction transistors (BJT) in their ideal form may be represented as ICIS. Field effect transistors (FET) may be represented by the VCIS model.

Complete amplifier model

The four ideal controlled-source models are used in representing various linear signal amplifications functions. We will focus on the voltage amplifier model because of its widespread usage and because some of its parameters are similar to those used in other circuits as well.

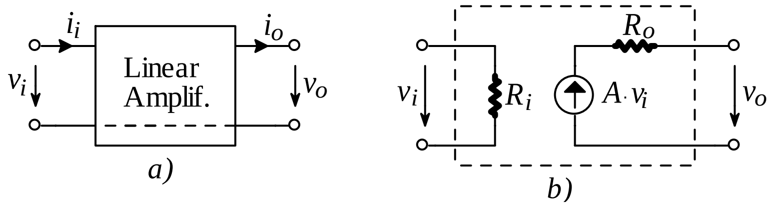

Figure – *Block diagram of a linear amplifier; a signal model utilizing a VCVS*

The signal model represented in figure can be used to represent a wide variety of complete amplifier circuits. We will assume that all passive parameters are resistive in this simplified model. The effects of reactive elements will be considered later.
It is customary in discussing complete amplifier models to use the general term impedance for the input and output effects, even when the resistance is the only parameter assumed.

Input impedance

The input impedance is the effective impedance across the two input terminals as “seen” by a signal source. For the resistive case:

$$R_i = \frac{v_i}{i_i}$$

The input impedance is important in determining the fraction of signal voltage that actually appears across the amplifier input terminals when the input signal source has an internal resistance.

Output impedance

The output impedance is the impedance portion of the Thevenin (or Norton) equivalent circuit as viewed at the output terminals.
The output impedance is important in determining the change in output signal with an external load (connected to the output terminals).

Voltage gain

With no load connected across the output, the output voltage is A times the input voltage. Thus, the open-circuit voltage gain is readily determined from the circuit diagram to be A. Under loaded conditions, the voltage gain will be reduced, as will be demonstrated later.

Cascade of amplifier stages - Input and output loading effects

The voltage v_i at the amplifier terminals can be expressed in terms of the open-circuit source voltage v_s and the voltage division between the source impedance R_s and the amplifier input impedance R_i:

$$v_i = \frac{R_i}{R_i + R_s} v_s$$

The output voltage v_o can be expressed in terms of VCVS Av_i and the voltage division between the amplifier output impedance R_o and the external load resistance R_L:

$$v_o = \frac{R_L}{R_L + R_o} v_s$$

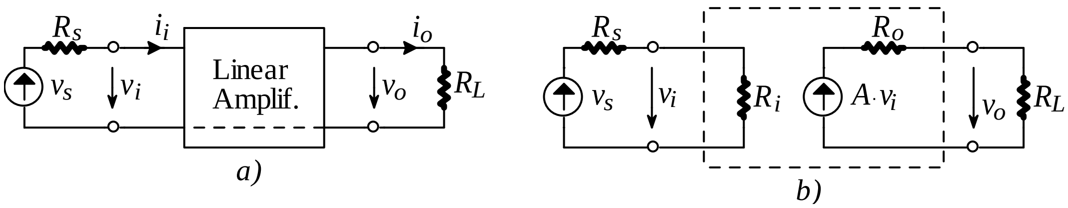

Figure – *a) Connection of an amplifier to a signal source and load; b) the linear signal model*

The net voltage gain from the source open-circuit voltage to the output load is:

$$A_s = \frac{v_o}{v_i} = \left(\frac{R_i}{R_i + R_s}\right) \text{x } A \text{ x} \left(\frac{R_L}{R_L + R_o}\right)$$

$$A_s = \begin{pmatrix}\text{Input}\\ \text{loading}\\ \text{factor}\end{pmatrix} \text{x} \begin{pmatrix}\text{open-circuit}\\ \text{voltage}\\ \text{gain}\end{pmatrix} \text{x} \begin{pmatrix}\text{output}\\ \text{loading}\\ \text{factor}\end{pmatrix}$$

The input loading factor is the voltage divider relationship reflecting the fraction of the source voltage appearing across the amplifier input terminal. The output loading factor is the voltage divider relationship reflecting the fraction of the open-circuit output voltage appearing across the load resistance. The open-circuit voltage gain in the ideal gain case would be achieved with no loading at either input or output.

Decibel gain computation

The basis for the decibel measurement is that of a power comparison. The input signal deliver a power P_i to the amplifier and the amplifier delivers an output power P_o to the external load. The power gain is defined as:

$$G = \frac{P_o}{P_i}$$

The decibel power gain G_{dB} in decibels is defined by:

$$G_{dB} = 10 \log_{10} G = 10 \lg \frac{P_o}{P_i}$$

If one assumes that the input impedance of the amplifier and the load resistance has the same value:

$$G_{dB} = 10 \lg\left(\frac{v_o^2}{R_L} \frac{R_i}{v_i^2}\right) = 10 \lg\left(\frac{v_o}{v_i}\right)^2 = 20 \lg \frac{v_o}{v_i} = 20 \lg A$$

Thus, a factor of 10 appears in decibel computations involving power gain and a factor of 20 appears when voltage and current gains are used. It is a common practice in electronics industry to define decibel gain even when the impedances are not equal; in this case the decibel ratio reflects not a power gain, but rather a voltage (or current) gain converted to a logarithmic basis. In this case a quantity A_{dB} will be defined as follows:

$$A_{dB} = 20\log_{10}\frac{v_o}{v_i} = 20\lg A$$

The conversion from decibel levels back to actual gain level:

$$A = 10^{A_{dB}/20}$$

Some useful values are given in the next table:

Positive dB values		Negative dB values	
A	A_{dB}	A	A_{dB}
$\sqrt{2}$ = 1.4142	3 dB	$1/\sqrt{2}$ = 0.7071	– 3 dB
2	6 dB	1/2 = 0.5	– 6 dB
10	20 dB	1/10 = 0.1	– 20 dB
20	26 dB	1/20 = 0.01	– 26 dB
100	40 dB	1/100 = 0.01	– 40 dB
1000	60 dB	1/1000	– 60 dB
2^n	$6n$ dB	2^{-n}	– $6n$ dB
10^n	$20n$ dB	10^{-n}	– $20n$ dB

Frequency response considerations

All linear circuits have frequency-limiting characteristics and it is necessary to apply frequency response analysis before a full treatment is possible.

Impedance

The complex impedance is defined as

$$\overline{Z} = \frac{\overline{V}}{\overline{I}} = R + jX$$

where R is the resistance and X is the reactance.

The three basic circuit parameters are resistance, capacitance and inductance. In steady-state ac circuit analysis, these parameters can be represented as function of radian frequency ω.

The radian frequency ω in radian/seconds (rad/s) is related to cyclic frequency f in hertz (Hz) by:

$$\omega = 2\pi f$$

Capacitance and inductance are represented by complex impedances:

$$\overline{Z}_C = \frac{1}{j\omega C} = \frac{-j}{\omega C} \;;\quad \overline{Z}_L = j\omega L$$

The capacitive reactance is negative and the inductive reactance is positive as follows:

$$X_C = \frac{-1}{\omega C} \;;\quad X_L = \omega L$$

In steady-state frequency response analysis ω (or f) is considered to represent any arbitrary frequency at which the response is desired and is treated as a variable. In this manner, the gain variations of an amplifier as a function of frequency can be investigated.

Transfer function

A linear circuit converted to steady-state form with the input and output represented as phasors can be used to define the steady-state transfer function $H(j\omega)$:

$$H(j\omega) = \frac{\overline{V}_o}{\overline{V}_i}$$

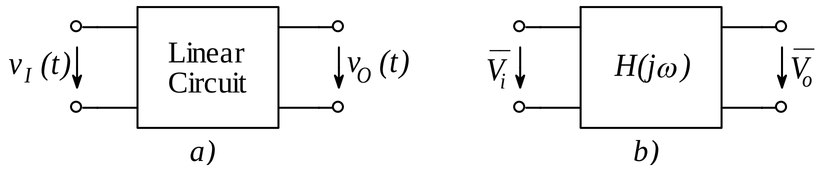

Figure – *A linear circuit: a) in the time domain, b) in the frequency domain.*

The steady-state transfer function (or simply "transfer function") is a generalization of the gain to the case where frequency-dependent elements are present.
Since $H(j\omega)$ is a complex function, it can be expressed in polar form. The amplitude (or magnitude) response $M(\omega) = |H(j\omega)|$ is the amplitude of the complex function and the phase response: $\varphi(\omega)$ is the phase shift of the complex function.

One-pole low pass model

A general method known as Bode plot analysis permits determination of amplitude and phase responses for rather complex circuits by some simplified graphical procedures.
The most common frequency response form arising in linear integrated circuits is a function that could be labeled in Bode plot analysis as the one-pole low-pass model.

RC low-pass circuit

To develop an intuitive feeling of the type of physical parameters that produce the one-pole response, the passive low-pass filter will be used.

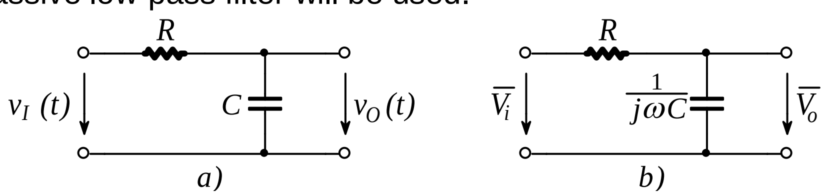

Figure – *RC circuit used to develop the form of a one-pole response*

Based on the voltage divider rule, the transfer function of the circuit can be computed:

$$\overline{V}_o = \frac{1/j\omega C}{R + 1/j\omega C} \cdot \overline{V}_i = \frac{1}{1 + j\omega RC} \cdot \overline{V}_i\,,\;\; H(j\omega) = \frac{\overline{V}_o}{\overline{V}_i} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\omega\tau}\,,$$

An important parameter of the circuit is its **time constant** $\tau = RC$.

Bode Plot

The amplitude and phase response can be derived from the transfer function:

$$M(\omega) = \frac{1}{\sqrt{1 + (\omega RC)^2}}\,, \qquad \varphi(\omega) = -\tan^{-1}(\omega RC)\,.$$

The point where $\omega RC = 1$ corresponds to the point where $M_{db}(\omega) = -3$ dB. This frequency is called the break frequency or corner frequency:

$$\omega_b = \frac{1}{RC} = \frac{1}{\tau}, \quad f_b = \frac{1}{2\pi RC}$$

The transfer function can be expressed in either of the forms:

$$H(j\omega) = \frac{1}{1 + j\dfrac{\omega}{\omega_b}} = \frac{1}{1 + \dfrac{f}{f_b}},$$

The magnitude and phase response will be:

$$M(\omega) = \frac{1}{\sqrt{1 + \left(\dfrac{\omega}{\omega_b}\right)^2}} = \frac{1}{\sqrt{1 + \left(\dfrac{f}{f_b}\right)^2}}, \quad \varphi(\omega) = -\tan^{-1}\frac{\omega}{\omega_b} = -\tan^{-1}\frac{f}{f_b}.$$

The amplitude response for different frequencies can be computed with the previous function:

- $f << f_b$, $M(f) = 1 = 0$ dB;
- $f >> f_b$, $M(f) = f / f_b = -20$ dB/dec;
- $f = f_b$, $M(f_b) = 1/\sqrt{2} = -3$ dB.

The amplitude and phase response are usually represented on semilog scales: the amplitude in dB (and phase in degree) on the linear scale as function of frequency on the logarithmic scale.

Such an example is presented in the next figure for a break frequency of 1 MHz (with both amplitude and phase response on the same graph – as provided by LTSpice simulator).

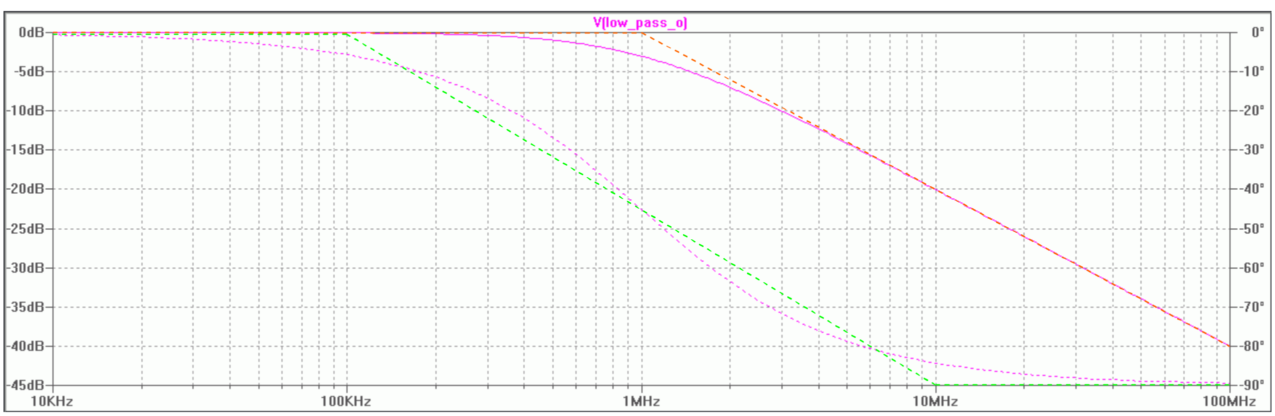

Figure – Amplitude (plain pink line) and phase (dotted pink line) response of an *RC low-pass circuit*

With Bode plot analysis the magnitude response is often approximated by the break-point approximation (the orange broken lines on the previous figures); it represents the first and second case previously computed (linear functions), extended to the break point. The bigger error is at the breaking point: $M(f_b) = -3$ dB.

A similar approximation procedure is used for the phase response (the green broken line), it is considered the following approximate function:

- $f < f_b/10$, $\varphi(f) = 0$ degree;
- $f_b/10 < f < 10 f_b$, $\varphi(f) = -45$ degree/decade;

- $f > 10 f_b$, $\varphi(f) = -90$ degree.

The biggest error is at the breaking points: $\varphi(f_b/10) = 90 - \varphi(10 f_b) = 5.7$ degree.

Extension to amplifiers

Many complex linear integrated circuits have frequency response functions of the general form of the *RC* circuit, at least over a major frequency range of interest. In such cases, the break frequency can be identified from appropriate measurements or specifications.

For an amplifier whose frequency response is dominated by a one-pole low-pass model the general form of the transfer function is:

$$H(j\omega) = \frac{A_0}{1 + j\dfrac{\omega}{\omega_b}} = \frac{A_0}{1 + j\dfrac{f}{f_b}},$$

except that the numerator is now A_0 instead of unity, a result of the amplification.